ELEMENTS OF MODERN LOGIC

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ELEMENTS OF MODERN LOGIC

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PREFACE

In this book I have endeavoured to make a contribution to the teaching of Elementary Logic. More specifically, my aim has been, in the first place, to provide an exposition of the essential principles of Logic which will be of service to those who are approaching the study of the subject for the first time; and, in the second place, to show the "open door" leading to a serious study of "Modern Logic"—that is, of Logic as the subject stands at the present time. Any endeavour to carry out this twofold aim involves a definite point of view from which the problems of "Modern Logic" may be estimated; and a brief statement of the author's point of view seems to be called for in this Preface.

It has been said that "a science which hesitates to forget its founders is lost." The reference was to what is commonly called "Aristotelian Logic." There is a body of traditional doctrine in Logic, which had its source in Aristotle. In the course of its history it has suffered from an increasing elaboration of formal technicalities for which the founder is not responsible. Nevertheless, a large part of the traditional Logic has entered so deeply into the structure of western thought, that some knowledge of it seems required even from the point of view of a liberal education. It may be admitted that the essentials of it can be adequately expounded, for beginners, without reference to their Aristotelian fountain-head—beyond what is necessary for the explanation of a few technical terms and methods. But this exposition must be *accurate;* and this condition I have carefully endeavoured to satisfy. I

am convinced that the really essential parts of the logical tradition founded by Aristotle are not obsolete. Still less are they false. But they cover only a part of the field, and that in an inadequate manner. In particular, the Aristotelian classification of propositions is seriously defective in taking account only of two-term propositions expressing the subject-attribute relation; and I have indicated the points at which the need for a wider view of Logic is evident, and the direction of the investigations based on the wider view.

Nevertheless, the syllogism, the distinctive form of inference derived by Aristotle from his analysis of propositions, "is *practically* important because it represents the form in which persons unschooled in logical technique are continually arguing; it is *theoretically* important because it exhibits in their simplest guise the fundamental principles which underlie all demonstration whether inductive or deductive; it is *educationally* important because the establishment of its valid moods and the systematisation of its rules afford an exercise of thought not inferior and in some respects superior to that afforded by elementary mathematics." This statement of the late Dr. W. E. Johnson I believe to be as wise as it is true; and since the mere reading of theoretical rules and discussions is as useless in Elementary Logic as it would be in Arithmetic or Algebra, I have added ample collections of carefully-selected exercises and problems in the body of the book and at its conclusion.

The observation that a science must not be pre-occupied with the work of its founders is applicable not only to the traditional Logic. The same observation must be applied to the Logic of "Induction" or Scientific Method. It would, of course, be absurd to regard John Stuart Mill as the "founder" of "Inductive Logic," which he never

claimed to be; but his *System of Logic*, first published ninety years ago, has so dominated even the elementary treatment of "Inductive Logic" that many Examiners seem unable to set a Paper in the subject without reference to some of Mill's views, and the current textbooks are largely occupied with exposition and criticism of his statements. The result is that students, who have yet to learn what science is or means, and who may never have seen even the simplest scientific experiment, have to find their way to the understanding of Scientific Method through learning criticisms of some of Mill's most carelessly-written arguments. The educational effects of this have been disastrous. Mill's great work is a classical statement of the Logic of Empiricism; and for that very reason it is essentially a book for the more advanced student. Yet, as if by some strange fatality, the conventional treatment of Induction directs the beginner's attention almost exclusively to the most doubtful and disputable parts of Mill's system.

In this book I have endeavoured to explain the nature of Science, first by showing how its methods grow out of the inductive interpretations of our experience which abound in common life. I have then made an extensive use of examples and illustrations in order to describe these methods in their distinctively scientific forms and to analyse their logical character.

In the preparation of the book I have received many useful comments and suggestions from Mr. H. E. Walsh, B.A., London; and these have enabled me to make the book more serviceable to the beginner. I hope that it will meet the needs of students preparing for any University Entrance Examination in this subject, and in particular for the Matriculation Examination of the University of London. The plan of the book has not admitted of the

discussion of certain topics—in particular, the diagrammatic representation of Propositions, and the theory of Probability—because any adequate treatment even of the foundations of these subjects goes beyond the range of a work intended for beginners. On the other hand, the references to the literature of the subject have been so arranged as to afford practical guidance to the student who desires to pursue the study further.

Passages which are printed in smaller type may be omitted by students who are reading the subject for the first time; but they must be included in a second reading.

Among my extensive obligations to other writers, extending over many years, it is a pleasure to acknowledge indebtedness to Mr. H. W. B. Joseph, *An Introduction to Logic*, second edition, 1916; to the late Dr. W. E. Johnson, *Logic*, 1921; to Professor A. Wolf, *A Textbook of Logic*, 1930; and to Professor L. Susan Stebbing, *A Modern Introduction to Logic*, second edition, 1933. For the contents of the following pages the author alone is responsible.

S. H. M.

LINCOLN.

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CHAPTER I

BELIEF AND REASONING

1. Characteristics of Belief

Experience shows that as life goes on, we are not only inwardly formed and changed by the accession of habits, but our minds are also enriched by a multitude of beliefs and opinions, and that on a variety of subjects. It is in fact a law of our nature that "beliefs" must take definite shape in our minds and acquire some degree of permanence. The importance of this fact is registered in the variety of words, akin in meaning with "belief," found in the English language—"assurance," "confidence," "conviction," "trust," "persuasion," "faith." Belief, so to speak, has ballast and anchor-cable. When it is formed, it is, for a time at least, a settled fact of the mind.

Now beliefs are not produced only by reasoning on the part of the person holding them. A belief may have many strong reasons in its favour; but we are constantly forming beliefs which we hold with a tenacity out of all proportion to the reasons which we can adduce for them. It may be that when we first admitted them, we had some kind of reasons, slight or strong, recognised or not, for doing so, and we have long forgotten them. We still hold the belief, though we have forgotten what the warrant was. Or, again, belief may die out with no tangible reasons sufficient to account for its failure; other causes, arising out of our condition, age, company, occupations, fortunes,

are at the bottom. Or, again, in spite of strong and even convincing arguments, real belief may never arise, as when men are loud in their *admiration* of truths which they show no sign of *believing*.

Thus a belief, once established, no matter in what way, has a certain mental stability. Let the reader call to mind anything which as a matter of fact he *believes*; let him imagine the belief destroyed by a word of criticism or a breath of opposition: the conclusion would be that he had not really believed it at all, and had only deceived himself into thinking that he did so. In complete belief the mind cannot entertain the idea of the contrary, even as a possibility. To believe the contrary would feel like saying that "black is white." "I beseech you, by the mercies of Christ," said Cromwell to the Scottish army on the eve of the battle of Dunbar, "think it possible you may be mistaken." But they could not. Herbert Spencer once proposed, as a test of truth, the "inconceivability of the opposite." It is not a test of *truth*; it is a test of a mentally complete *belief*, which may be a delusion. In fact, the easiest examples to give of such complete belief, would be from the various types of delusion. But without entering on this field at all, we should all be willing to confess to the holding of some beliefs so deeply rooted in our minds as to make any contrary assertion seem incredible. Through years of personal intercourse with a friend, and experience of his life and habits, I have come to so strong a conviction of his character, that when he is charged with certain conduct I find it impossible to believe in his guilt. This is *mental certainty*, understood as a state of the mind, whatever the rational grounds for the certainty may be, or whether there are such grounds at all.

We say, "I believe . . . ," and if our statement stops there, the natural question is, "*What* do you believe?"

Any reasonable answer to this question brings out another fundamental aspect of belief—namely, its claim to be true of reality. When the meaning of a belief is expressed, it takes the form of an assertion or judgment. Judgment is the attitude of the mind towards a suggested fact, when it affirms or denies, accepts or rejects, the suggestion. We accept the suggestion as being true of reality, or we refuse it, put it away. This latter act of the mind is usually called "disbelief"; but "disbelief" is a kind of belief which, in reference to the particular suggestion, has a negative meaning. "I do not believe that my friend is capable of such conduct." This is the expression of an actual belief that his character excludes the possibility of it. The state, which is mentally the opposite of belief, whether the belief expresses itself in affirmation or denial, is *doubt*.

2. How Beliefs are Produced

It follows from what we have said that there are various ways in which beliefs may be produced; in other words, that there are various causes of belief. And not all these *causes* are *reasons*; some of them may be, but not all. For our present purpose, the most important ways in which beliefs are produced may be classified as follows: (1) Perception and Introspection; (2) Memory; (3) Imagination; (4) Suggestion; (5) Communication; (6) Reasoning. We will look briefly at each of these, before concentrating on the last-named factor, which leads into the heart of our present subject.

(1) We have experience of a world of real objects; we apprehend them in sense-perception, and some of their qualities are revealed to us in what psychologists call the "sensations" of our outer senses. The statement that the

foundation of our belief in the reality of things is the resistance they offer to our endeavours, is strictly true. In the case of a material object, the strongest evidence of its reality is the resistance offered by it to our bodily efforts to move or change it. Still more convincing is the active varied resistance to our manipulations offered by other persons and by animals.

It is not of course intended to suggest that, in the case of belief based on sense-perception, the reality which is the object of belief can be defined simply as " resistance " or " opposition." We apprehend the outer world not only through the sense of touch. Sight and Hearing are called the " higher senses " because of the wealth of information which they provide about it. " Seeing is believing," says the proverb.

Introspection may be classed with Perception as a mode of observation; the latter being directed to the outer world, the former to the mind's own processes. We thus become aware of the conscious aspects of our mental life; and the chief difficulty of introspection is to secure that the facts shall be apprehended as they are " in themselves " —in other words, independently of the effects of our own endeavour to observe them.

(2) Memory is the foundation of our belief in past events, and the order in which they entered into our experience. Memory is always limited in its range and liable to error, but this does not invalidate the fundamental fact of its direct contact with reality. Suppose that I am asked, as witness in a Court of Justice, " Did you see the prisoner on such and such a day? " I may remember that I did see him. If my statement is challenged in cross-examination, I may confirm it by recalling events on the date in question, together with the prisoner's movements and my

own, and finding that they all hang together in a coherent sequence; my confidence in the truth of my original statement is only strengthened. This illustrates the production of belief in the reality of a past event by memory, and the strengthening of the belief by further acts of memory.

(3) By Imagination, as a source of belief, we mean the play of imagery, visual or other, and its building up into forms which are not copies or reproductions of past experience, and *in this sense* are " unreal." So far as Imagination is subject to the control of our own conscious volition and purpose, so far it has no tendency to pass into or create belief; on the other hand, so far as it escapes such control, so far it tends to create belief.

For an extreme case, take that of the patient under an hallucination. In an early stage of his trouble, he may dismiss by an effort the phantom figure or the voice whispering of threats or persecution; and so long as he can do this, he does not believe them real, in spite of their sensory vividness. But in a more advanced stage of the disorder, he cannot dismiss them; and then he begins to yield himself helplessly to belief in their reality. Or, again, when a man in a normal state of health misinterprets the evidence of his senses, and his interpretation is a spontaneous act inseparable from the sense-impression, he accepts it as given, and believes it accordingly. In like manner, an acquired mental bias may obtain such a hold on the mind as to mould the evidence of a man's senses, or any other evidence, without his conscious intention; so that he believes what he wants to believe or what he hopes for or desires, or on the other hand, what he dreads. The object hoped for or desired or dreaded appears to be given as real. On the other hand, we must remember that beliefs produced by Imagination are not always delusions,

superstitions, or errors; an "acquired mental bias" may be well founded.

(4) Suggestion, as a cause of belief, is the process in virtue of which beliefs are directly induced by a kind of mental influence or force, independently of logical evidence, or reasoning to the conclusion; although the absence of reasoning or rational persuasion does not necessarily imply that the belief is wrong. It is true that in extreme cases Suggestion passes over into the region of the abnormal, as when the suggestions of the hypnotiser produce actual hallucinations in the hypnotised person. But there is a broad region of normal familiar fact, where we can observe the production of beliefs by Suggestion without reasoning. Much of the art of advertising, and many methods of propaganda in politics and other important subjects, are of this kind. Everyone is "suggestible" to some extent—some much more so than others. Suggestibility, for example, makes a man susceptible to all kinds of social influences in the formation of his beliefs and the determination of his conduct.

(5) In mentioning Communication among the causes of belief, we refer to statements made by others, and made in order to be believed, which we do believe without personal verification, because we are willing to accept their trustworthiness.

It is evident that to Communication, so understood, each of us owes by far the larger part of the beliefs which constitute the working capital of his mind. It covers the process of our education in the widest sense, including everything which we accept through reading or hearing, and every occasion in the course of our lives where statements made by others—and made, we repeat, in order to be believed—are accepted by us and believed. It is

practically impossible for us to verify personally more than a small part of what is accepted in this way. The conclusions of history and physical science abound in illustrations of this.

Suppose, for example, what sometimes happens, that we meet with a person who denies the roundness of the earth. So far as he can observe it, it appears to be flat; and therefore he believes it to be flat. Most of us at once reject his pretensions, without any attempt to verify the roundness of the earth, because of our confidence in a vast work of constructive interpretation which has been built up by the co-operative thinking of many generations, on which has been based all that we have been taught about the earth and its relation to other bodies in the heavens.

In many of the familiar pursuits of everyday life, where personal verification would be possible, there is neither time nor reason for anything of the kind. The clerk, said Carlyle, cannot be always testing his "ready-reckoner." True, he finds by using the ready-reckoner that he does not go wrong,—it "works" or is verified by the results of using it. The same may be said of some of the conclusions of applied science; they admit of verification apart from our understanding of the methods by which they are reached, because we can observe their working. The astronomical information given in the predictions of the almanac—the hours of rising and setting of the sun and moon, and of high and low tide, the eclipses of the sun and moon, and so forth—are an effective illustration of this practical verifiability. Verification of this kind introduces us to Reasoning as a way of producing beliefs.

3. Reasoning

We now come to the factor in belief with which Logic is specially concerned: belief as produced by reasoning. It

will be convenient henceforth to use the word "judgment" instead of "belief," since we are speaking of belief as involving *assertion*—affirmation or denial. The process of reasoning is essentially the combination or "synthesis" of judgments already held; and these when combined are found to warrant a new judgment which could not have been derived from any of them separately. The data combined are called the *premisses* of the inference, and the new judgment warranted by the combination is called the *conclusion*. But in order that the data combined may yield any conclusion, they must have something in common; this is, so to speak, the pivot on which the inference turns. The following is a simple example.

A traveller walking alone on a moor has lost all his bearings, and does not know which way to turn, when suddenly on the far horizon he catches sight of a hill of a peculiar shape, which he recognises; and he knows the way from that hill over the distant rising ground to his destination out of sight beyond. Here we have two data: the way from here to the hill (perceived), and the way from the hill home (remembered). These premisses have a common factor, the hill, and this common factor warrants the conclusion, "Now I know my way home." If he had mistaken the shape of the hill, there would have been no real common factor and no valid inference. He would still have been lost. And even with the recognition of the hill, giving one premiss, there could have been no inference until the second premiss, the recollection of the way home from the hill, had been added.

We have said that Logic is specially concerned with reasoning. This does not mean, with the reasonings which as a matter of fact do occur in people's thinking, but with the question, What are the best and soundest kinds of reasoning? The essence of the process of reasoning we

repeat—is the synthesis of two or more judgments which when their content is explicitly stated involve a common factor, and by means of this common element the synthesis reveals a new fact which could not be derived from any of the original judgments taken separately.

We must remember that although elaborate and complicated chains of argument are by no means uncommon, an inference may on the other hand be extremely condensed in its verbal expression, as in the saying, "This is too good to be true." Moreover a whole process of reasoning may go on in the mind very rapidly and effectively, and yet may be all mentally "implicit," in other words, may not emerge into clear consciousness at all. When Robinson Crusoe saw the footprint on the shore, he started back in a state of complex and confused emotion. But this was not a response to the mere visual impression; it was his reaction to a process of reasoning by no means simple in character, which passed almost instantaneously through his mind. The first step may be expressed thus: "All prints of such a kind are made by men; this is a print of such a kind; therefore this was made by a man." In like manner, the second step may be expressed: "I have not been here before, therefore this was not made by me but by some other man." And so forth, until the disquieting conclusion is reached that the unknown man may be somewhere at hand.

When we take into account what is implicit in any process of inference, we shall find, by the help of some typical examples of the process in its simpler forms, that three kinds or types may be distinguished; and in order to illustrate them, we shall also distinguish between what at this stage of our study may be called "particularised" judgments and "universalised" judgments. A "particularised" judgment refers to a particular individual or fact

or case or to a number of such cases; a "universalised" judgment seeks expression in a statement beginning with "all" or some synonymous adjective or adverb: for instance, "All animals are mortal."

We find then that reasoning or inference may be (1) from particular to general; (2) from particular to particular; (3) from general to particular. Most of our reasonings are of a composite character, in which more than one type can be traced. The few examples which we shall give should be regarded as typical specimens, like botanical specimens for dissection.

(1) Consider the following as an illustration of the first type of reasoning mentioned above:—

Yesterday it rained in the evening;
All yesterday the smoke tended to sink;
Therefore smoke-sinking may be, or is sometimes, a sign of rain.

We have here two particularised judgments, and a common factor ("yesterday," "all yesterday") connecting them; and the conclusion is a suggested generalisation. As it stands, however, it is little more than an observation and a guess. In such arguments, we attempt to generalise from one or more particular cases. To generalise in this way seems to be a fundamental tendency of our rational nature. The process is necessary, and justifiable, provided we bear in mind the caution that generalisations based on "a number of particular cases" require further investigation and testing before they can be accepted as valid conclusions. The instances which we have in view serve to raise the question: Is there a real connection between the two qualities or characteristics which we have noticed in each of them (smoke-sinking and rain), or is the combination merely accidental?

Let us now carry the argument a step further by modifying our statements as follows:—

Smoke that goes downwards is heavier than air;
Particles of moisture are heavier than air;
Therefore particles of moisture may be in the descending smoke.

This again is of the type (1). As it stands, it is inconclusive; because the smoke may be sinking for some reason which has nothing to do with particles of moisture. But it affords a tentative justification of the generalisation originally suggested; it assigns a possible cause by bringing forward an analogous case—a cause which would naturally act in the way suggested.

Arguments of this kind are of great importance in practical life. It is true they may be so inconclusive as to be simply silly if put forward seriously: for instance:—

Fever-stricken persons are excessively thirsty;
This person is excessively thirsty;
Therefore this person is fever-stricken.

Here we have an attempt to argue from a symptom, which may have quite other causes. Nevertheless, when we have a number of independent symptoms, all suggesting the same conclusion, we regard the conclusion as, for practical purposes, certain. A medical "diagnosis" is really an argument of this kind. We shall see, in the sequel, that all arguments from "analogy" are fundamentally of this type.

In both these examples, we have a generalisation suggested by the particular cases in view; in the one example, by enumeration of one or more instances, in the other, by explicit comparison of instances. We pass to a general principle or law illustrated in them. As we shall see,

when we pursue our study, reasoning of this first type corresponds to what logicians call "inductive generalisation." The tendency to inductive generalisation is fundamental, and is found in operation at all stages of mental life.

(2) We distinguished a second type of reasoning as "from particular to particular." Here the interest lies in the particular cases, and we pass directly from one to another without consciously forming in our minds any generalised judgment.

The most familiar quantitative or measurable aspects of experience produce arguments of this kind. I weigh two objects in succession against a pound weight in the scales, and they exactly balance it: their weight is therefore the same—one pound. In abstract terms, "A and B are equal to the same thing (C), therefore they are equal to one another." A girder, stated to bear a load of twenty tons, is tested to thirty tons without injury: therefore it can safely be used in a bridge bearing only ordinary wheel traffic. In abstract terms: "A (the tested resisting power) is greater than B (stated resisting power), B is greater than C (required resisting power), therefore A is greater than C." Relations of time and space likewise produce many examples: "I was not more than three years old when it happened, for it was some time before we removed to Liverpool, and I can remember my fourth birthday was after that." It will be seen that these inferences are based on *serial order* in space, time, quantity; but inferences of this kind are by no means limited to cases where the reasoning depends on the place of the common factor in a serial order. In fact we are constantly reasoning from one particular case to another *without explicit generalisation*.

Although such inferences may be described as "from one particular case to another," the particulars are *not*

unrelated. They must have a common factor if inference is to be possible. As a rule, the common factor is explicitly stated. The example of the lost traveller, given above (page 8), illustrates this. We have distinguished the second type of reasoning as a separate type, because of its great practical importance. We shall see, however, at a later stage of our inquiry, that it is not at bottom an independent type of reasoning: because when it is carried far enough and analysed, it becomes an argument either " from general to particular " or " from particular to general."

(3) The third type of reasoning, which we distinguished above, was described as " from general to particular." Whenever we apply previous knowledge to a given case, we are reasoning in this way. We may, for example, be puzzled by the liability of thick glass to crack more easily than thin glass when heated (except in the case of glass specially manufactured to resist heat). Stated formally and, comparatively speaking, fully, the relevant reasoning would be this. Whenever material substance is heated, it expands; and glass, being a material substance, expands when heated. All hotter substances expand more than those which are less hot. When thick glass is heated, the surface is (at first) hotter than the interior, hence the surface expands more than the interior. Here we see the successive applications of generalisations, already formed and accepted, to a given case about which a question has arisen.

If we turn again to the inquiry supposed to be raised in the examples already given, as to the connection between smoke and rain, we may sum up the result of it in an inference " from general to particular," in some such form as this:—

All particles which sink in the air in damp weather more than in dry are loaded with moisture when they sink ;

Smoke which descends before rain is an example of particles which sink in the air in damp weather more than in dry;
Therefore smoke that descends before rain is loaded with moisture when it descends (i.e. *is really connected with the cause of rain*).

Inference of this kind is of the greatest importance in science and in practical life. Science seeks for results which are "laws"; that is, which are statements universally true about certain kinds of fact; and every time we explain a fact by the law, every time we find a new exemplification or application of the law, we have an inference of this third type.

The reader will observe that in this chapter we have, so to speak, brought forward specimens of the kind of question with which Logic is concerned in the study of reasoning. Before proceeding further, we must give a preliminary statement of the range or scope of this study.

4. Provisional Definition of Logic

We are now in a position to give a definition of Logic, which will be sufficient to serve as a starting-point for our inquiry. *Logic is the systematic investigation and study of the principles of valid reasoning; in other words, of the principles on which the validity of reasoning depends.* The most important words in this definition are "valid" and "validity," the adjective and the noun. It will be enough to consider the adjective. "Valid" means primarily *self-consistent.* We say "primarily" because as we shall see, "validity" means more than self-consistency. To think rightly we must think not only consistently but truly. None the less, "valid" reasoning must at least

be self-consistent. A statement or an argument is self-consistent when it so hangs together that thought may pass through it, as it were, from beginning to end without falling into contradiction with itself by the way. The statement, "Some squares are round" violates this fundamental requirement; so does the argument, "All men are fallible, all Judges are men, therefore some Judges are not fallible." But consistency does not concern only the contents of single statements or short arguments. "The diligent reader," it has been said, "may discover on different pages of a connected treatise statements which no charity can construe as inter-consistent; the statements may be separated by more than a hundred pages, but the requirement of inter-consistency will still compel such a logical re-adjustment of the passages as will make them maintainable by one and the same thinker in one and the same discourse."

The definition speaks also of the "principles" on which the validity of reasoning depends. These principles can be discovered only by investigating the methods of reasoning as we find them in the various sciences and other departments of human study, and in ordinary discourse and thinking. But it is not the business of Logic to discuss the particular problems dealt with in any branch of knowledge or science.

The usual treatment of Logic lays out the subject in two branches corresponding broadly to the distinction, which we have illustrated, of reasoning "from general to particular" and "from particular to general." The first branch of Logic is called "Deductive Logic," and the second branch is called "Inductive Logic," "Deductive Logic," which is also called "Formal Logic," studies the principles of *self-consistency* in thinking; and "Inductive Logic," which sometimes is called "Material Logic,"

studies the principles of consistent thinking in a wider sense—the principles on which its "consistency with reality" depends. We shall begin therefore with Deductive Logic, and, in the first place, we shall discuss the constituents and the classification of propositions.

CHAPTER II

PROPOSITIONS AND TERMS

1. Proposition and Judgment

In the previous chapter we have distinguished reasoning from other sources of belief; and we have brought forward examples of inference in some of its simpler forms. We have seen that the process of reasoning is essentially a combination or synthesis of judgments which, when their content is explicitly stated, are found to involve a common factor; and by means of this common element the synthesis reveals a new fact which could not be derived from any of the original judgments taken separately. For our present purpose, in this book, we shall understand by "judgment" the attitude of mind towards a suggested fact when it affirms or denies, accepts or rejects, the suggestion. Judgment is the mental act of assertion, affirmative or negative. It admits of being characterised as true or false; otherwise it is not a judgment in the logical sense. Thus, wishes, commands, questions, and exclamations are not logical judgments. We cannot say of mere "wishes" that they are true or false, though they may be reasonable or unreasonable; and "commands" call for obedience, not belief; they do not state truths or claim to do so.

Judgment may be called the *unit of thought*; for all our deliberate thinking consists in making statements or assertions; and if we are to have truth or falsity we must at least have a judgment.

For the purposes of logical treatment, every judgment must be expressed in words. Thought is prior to verbal

language; but thought can make no progress without embodying itself in language. There is a two-sided or reciprocal dependence between them. We may illustrate it thus: An army may over-run a country, but the country is only conquered by the establishment of fortresses; words are the "fortresses" of thought. Or better, thus: In tunnelling through a sandbank it is impossible to proceed until the present position is made secure by an arch of masonry. Words are such "arches" for the mind. In Logic we are interested in words only as the visible and audible forms in which thought fixes and controls its own meaning.

A **Proposition,** in Logic, *is a Judgment in an intentionally fixed verbal form.* Unless it is so expressed, the Judgment is not available for logical purposes. In other words, Judgment and Proposition are inseparable; and we may define a Proposition as *a statement which is capable of being either true or false:* usually, a statement in words; but as we shall see, propositions may be expressed in symbols which are not words.

Now consider any simple proposition, such as, "This rose is yellow," or "All men are fallible," and examine its structure. It contains two principal *constituents* or component factors, and a third constituent which in these examples is expressed by the present tense of the verb "to be," and is called the "relating factor." The constituents, other than the relating factor, are called **Terms.** *A term, therefore, is a word or group of words capable of forming a constituent of a logical proposition.*

2. Classification of Terms

When we take to pieces a chain of reasoning, we find that it consists of propositions; and we take to pieces a

proposition, which is expressed in logical form, we find that, in addition to the relating factor, it consists of "terms." A term, as we have seen, is just a potential Subject or Predicate, whether or not it is doing actual service within a proposition. It is often convenient, when we are considering a term apart from a proposition, to speak of it as a "name."

It has long been recognised that names have different logical characters, and can be classified accordingly. The distinctions which have been made are not all equally important from the logical point of view. They are as follows:—

(1) General and Singular names.
(2) Connotative Singular names and Non-connotative Singular names.
(3) Abstract and Concrete names.
(4) Positive and Negative names.
(5) Collective and Non-collective names.

(1) To understand the logical character of **General Names**, we must illustrate the formation of the idea of a *class*. For example: very early in the history of astronomical observation, long before instruments were available, the difference between "planets" and "fixed stars" was discerned. Let us imagine some astronomers in ancient Babylonia considering the well-known heavenly bodies afterwards called Jupiter and Sirius, and comparing them. It was observed that they agree in being small, bright, shining bodies which rise and set and move round the heavens apparently with equal speed. By minute examination, however, it was noticed that Sirius gave a twinkling or intermittent light, whereas Jupiter shone steadily. More prolonged observation showed that Jupiter and Sirius do not really move with equal and regular speed,

but that the former changed its position in the heavens from night to night in no very simple manner. When the comparison was extended to others of the heavenly bodies, it was found that there were a multitude of stars which agree with Sirius in giving a twinkling light and in remaining fixed in relative position to each other, whereas several other bodies were seen which resembled Jupiter in giving a steady light, and also in changing their position from night to night among the fixed stars. Thus, by bringing together mentally a number of objects which agree, was formed the general idea of *fixed stars*, while from several other objects in like manner was formed the general idea of *planets*. This illustrates in a simple case the formation of the idea of a "class." A class is simply the (indefinite) number of individual objects or cases characterised by the fact that each possesses a certain group of qualities.

The name of a class is said to be a *general* or *common* name because it is applicable, without change of meaning to each of a number of objects which resemble one another in some characteristic features or aspects, called in Logic *attributes*. Now examine the meaning of any typical general term, such as "man." We see at once that it has a two-fold meaning, or rather, two kinds of meaning:—

(*a*) It stands for a whole group or class of beings which it names and distinguishes from other groups, and the name is applicable to each member of the group: these objects, or *beings*, tō which it is applicable constitute what is called the *denotation* of the name.

(*b*) It also stands for the attributes which these objects or beings have in common and which distinguish them from other groups—in this case, the attributes constituting "humanity": these attributes constitute what is called the *connotation* of the name.

As regards the denotation, the class denoted need not be numerically definite or limited; it is *known through the attributes*, and any instance of these, whether we are aware of its existence or not, constitutes a member of the class.

It has been objected that in names such as "unicorn," "dragon" we have connotation, but the objects referred to do not exist, and therefore we have no denotation. To this it has been replied (as by the present writer, *Introductory Textbook of Logic*, Chapter II., page 18; Chapter IV., pages 108-9) that by denotation we do not mean only existence in "the real world"; existence in any kind of world which is being spoken of as the subject of discourse is sufficient, e.g. the ideal world, or the world of heraldry or folk-lore, or any of the various worlds of belief or imagination. We believe that this view is in substance sound: but in stating it, it is better to avoid such highly metaphorical expressions as "many kinds of worlds." Some writers speak of different "universes of discourse (or reference)." It is better to speak of the *context within which a term is applied in any given proposition*. In this sense the term "Universe of Discourse" is admissible. The context may be, for example, that of human mythology, that is, of *human beliefs* which are part of the real world. The term "fairy" has connotation; it stands for a group of characteristics. It has no denotation, if denotation means existence in the real world in the sense in which cows and sheep exist. But the characteristics of a "fairy" are or have been human beliefs, and these beliefs exist as part of the real world; and the characteristics of a "cow" are also human beliefs, and these beliefs are part of the real world. The difference is that in the latter case the beliefs are warranted by perception and observation, and in the former case they are not. The denotation of "cow" is a matter of observation, while that of "fairy" is a matter of mere belief. (See W. E. Johnson, *Logic*, 1931, Volume I., pages 161-6; and L. S. Stebbing, *A Modern Introduction to Logic*, second edition, 1933, pages 55-6.)

A name which has these two kinds of meaning is said to be *connotative*. In other words, a name described by the *adjective* "connotative" has a two-fold meaning: (*a*) a group of objects, *together with* (*b*) an attribute or attributes which they possess in common, and on account of which they are called by that name. A *non-connotative* name is one which lacks one side or the other of this two-fold meaning.

From general names, understood as above, we must carefully distinguish what are usually called **Singular Names**—although "individual names" would be a more appropriate designation. A "singular name" is the name of one unique individual to the exclusion of every other. The name, as long as its meaning does not change, denotes only this single object.

(2) In the next place we have to explain a very important division of singular names. Singular names may be divided into (*a*) uniquely descriptive names, and (*b*) ordinary Proper Names.

(*a*) A **uniquely descriptive name** is so called because it serves both to identify and describe some one object or individual. Such are: "the centre of the earth"; "the present King of England"; "the highest mountain in Asia"; and all descriptive names introduced by singular demonstrative adjectives such as "this," "that." "Man" is a general name; "that man" is a singular name. The logically Singular name must be distinguished from a name which is grammatically in the singular number. Any logically Singular name is, of course, grammatically in the singular number; but a name may be grammatically in the singular number and yet be logically a general name: "*a man's hand* is a wonderful structure." Uniquely descriptive names are connotative in the sense defined above. Such a name may be connotative in a high degree: "the smallest star visible to the unaided eye" (the Sun).

(*b*) A **Proper Name** is a name given to an individual—person, place, or other object—for purposes of identification. Its sole purpose is to indicate that one object. Thousands of men and boys, not to mention dogs, cats, or other animals, may be called "John"; but the name does not stand for anything which they have in common. In

each case it is given for identification. On the other hand, in each case it *acquires* a meaning by association; it comes to suggest many of the personal qualities and characteristics of the individual named. Now this acquired meaning or suggestion is different from the *connotation* of a general or class name; and in the sense in which we use the word, a Proper Name is evidently non-connotative.

The conclusion stated above has been contested by some logicians, including the present writer (*Introductory Textbook of Logic*, Chapter II., pages 33-8). But this involves the use of the word "connotation" in a sense wider than that which we are now adopting. In fact, the words "connotation," "denotation," and the corresponding verbs "connote" and "denote," involve important ambiguities. (See the references given in Stebbing, *A Modern Introduction to Logic*, second edition, 1933, pages 27-30.) Here we will only make the following observation. Let us compare (*a*) a uniquely descriptive name like "this great writer," descriptive of some particular individual whom we have in mind, and (*b*) a proper name like "John Smith," applied to some known individual and thereby acquiring a descriptive meaning (his personal qualities, etc.) by association The difference between the two cases is, that in (*a*) the descriptive factor, "great writer," is implied by the name before it is given, and in (*b*) it is acquired after the name is given.

There is a clear logical difference between (1) an ordinary Proper Name like "John Smith," (2) a uniquely descriptive name, as "the present King of England," (3) a general or class name, like "king." A Proper Name lacks one side of the two-sided or two-fold meaning (object *plus* attribute) which all general names and significant Singular names possess. On the other hand, the meaning which a Proper Name has acquired when used as a designation of a particular individual may be generalised and the name used as a type, in which case it becomes a general name: "a Don Quixote," "a Daniel—a second Daniel," a "Solon," "a Croesus," "a Nero."

Propositions having Singular terms as subjects are of

course constantly occurring, and are called "Singular Propositions."

(3) We now come to the distinction between **Abstract** and **Concrete Names.** An abstract name is the name of a quality, property, or action considered apart from any object to which it could belong; whereas a concrete name is the name of an object regarded as possessing attributes. Hence all class names, uniquely descriptive names, and Proper Names, are concrete. This distinction concerns the *use* of names, for some names may be used now as abstract, now as concrete. Hence before we can decide whether a given name is abstract or not, we must consider a proposition in which the name is used as a Term. Thus, "society" may mean an actual community of men living together in a certain way, and then it is concrete; or it may mean the *way in which* they live together, and may be thought of as apart from any particular example of it: then the name is abstract. As a rule, when a term is used abstractly there is no doubt about the fact. For example, consider the following propositions: "A tyrant's hate is a thing we need not fear"; "The hate of my fellows is a force I dare not face"; "A man's virtue is to be truly a man"; "The thoroughness of this work is remarkable." In these propositions, the primary emphasis is on the particular cases or instances in which the quality is manifested: "a tyrant," "my fellows," "a man," "this work"; hence the subject-term in each case is concrete. On the other hand, consider the following: "Hate is old wrathe" (Chaucer); "Virtue is a self-rewarding activity"; "Thoroughness is a test of efficiency"; "The unknowable is the unthinkable." In these the primary emphasis is on the quality apart from any particular case of it; hence the subject-term in each is abstract.

(4) The distinction of **Positive and Negative Names** has sometimes been used in a way which, though it is of practical importance, is not really a matter of logical concern. The distinction has been based on the presence or absence of a particular attribute. It is true that pairs of names occur in the English language, in each of which we find one name implying the presence, and the other name the absence, of a given attribute. Sometimes two different words are used to express the two implications; sometimes the negative name is formed from the positive by a prefix. The following are examples:—

Positive Names.	*Negative Names.*
(*a*) Hot.	(*a*) Cold.
(*b*) Gratitude.	(*b*) Ingratitude.
(*c*) Agreeable.	(*c*) Disagreeable.
(*d*) Manly.	(*d*) Unmanly.

The negative name does not imply mere negation, but the presence of some other quality; in each of the above instances the negative name implies the presence of an actual quality which is the opposite of the one excluded.

What is of logical importance, however, is not the presence or absence of any particular quality, but the nature of the opposition or contrast between the two names. In the examples given above, each pair of names belongs to a particular context or a particular class of things. "Universe of Discourse" is the awkward term which has been used. We shall use the less pretentious and less misleading term "field of reference." Thus, in (*a*) above, the field of reference is "temperature"; in (*b*), "habit of feeling"; in (*c*), "emotional quality"; in (*d*), "manner of behaviour."

A pair of names which stand for opposites but which do not between them cover the whole field of reference, are

called **Contrary Names.** Thus, to take again the examples given, there are other varieties of temperature beside those that are properly described as "hot" or "cold"; other kinds of feeling in response to benefits received than definite "gratitude" or definite "ingratitude"; other kinds of emotional quality than the "agreeable" or the "disagreeable"; other kinds of behaviour than the "manly" and the "unmanly."

The opposition expressed by contrary names is of various kinds. Some pairs of contrary names indicate things which stand furthest apart within the same class, as "virtuous" and "vicious," or "black" and "white." A more general case is that of *incompatibility*—the opposition of qualities which cannot be possessed by the same thing in the same way, as "round" and "square," "red" and "green," "one" and "many"; while "red" and "round," "large" and "square," etc., are entirely *compatible.* But Logic can give no general account of all the different kinds of opposition which pairs of contrary names express. Our understanding of the opposition depends on our knowledge of the things. In this sense contrary opposition is said to be *real* or *material* opposition.

On the other hand, there are pairs of names which together do cover the whole field of reference. Thus, "British" and "alien" cover the whole class of "persons" (in the legal sense); "male" and "female" cover the whole class of organic beings (above a certain elementary stage of development). These are called **Contradictory Names.** Sometimes, as in these cases, a pair of contradictories is expressed by two different words; sometimes the contradictory name is formed by prefixing "not-" or "non-" (with hyphen): "not-white" means every kind of colour except "white."

This kind of opposition is partly material and partly

formal. Provided we know enough of "colour" to know that "white" does not exhaust the whole of it, then we can form the contradictory name "not-white" to include all the other colours, even if we do not know what colours are included in it. Though partly formal, it is convenient to describe such pairs of contradictories as "material contradictories."

Finally, we come to a type of contradictory name which is purely formal, because it is made by the addition of the prefix "not-" or "non-" *without limitation to any field of reference*. Thus, "not-white," as a formal contradictory, means anything in the heavens, the earth, or the mind of man, which *is not* "white." Such a name is not the name of a class; we could never make a class of such a chaotic mass of the most different things, or even hold them together in our thought as one idea. If we try to think of the denotation of a formally contradictory term, we must think of "either this, or this, or this, or . . ." and so on indefinitely through everything which is not denoted by the original term.

(5) A **Collective Name** is one which stands for a group of individuals, persons or things, considered as acting together, or at least as belonging together, so as to form a single whole. Familiar examples are such names as "committee," "team," "army." The collective name is applicable only to the group as a whole, not to any individual member of it. All the players together constitute "the team," and no one member or smaller group of members is "the team." Here we see clearly the difference between a collective name and a general name or class name. The class name is applicable to each member of the group; the collective name is not. This fact is also expressed by saying that a name which is used as a class name is used *distributively*.

Collective names may be singular, as "the British Army now in India," "the present House of Commons"; or general, as "a committee," "a library." When a name may be used in both ways, the collective and distributive meanings must be carefully distinguished. Thus the name "committee" is used distributively as being applicable to each one of the many different groups formed in the manner which the name signifies. But as applied to any particular one of these groups, its use is not distributive but collective; it cannot be given to any member of the group, but only to the group as a whole.

Neglect of this distinction may lead to serious fallacies or mistakes in reasoning. The word "all," for instance, may be used either collectively or distributively, "all men" may mean "any man," or "all men together," *i.e.* the human race as a whole. And what is true of "all" distributively may not be true of "all" collectively; and *vice versa.* Two important types of logical fallacy arise from overlooking this distinction.

(i) We may attempt to pass from a statement about a class of things distributively, that is, about each and any separately, to *the same statement* about the whole considered collectively. This is technically termed the fallacy of *Composition.* It would be a fallacy of "Composition" to argue that because every member of a jury is very likely to judge erroneously, the jury as a whole is very likely to judge erroneously; or to argue that because any of the witnesses in a law case is likely to give false or mistaken evidence, no confidence can be placed in the concurrent testimony of a number of witnesses.

(ii) We may attempt to pass from a statement about a group as a collective whole to the same statement about each or any individual member of the group. This is

technically termed the fallacy of *Division.* The statement that a certain political party is a " blatant faction " does not justify the inference that the opinions of every one of its members are blatant and factious; to say that " the Germans are an intellectual people " does not warrant the conclusion that this or the other German is intellectual; to say that " this work is well done *on the whole* " does not mean that every detail in it is " well done "; and so on.

In the above, we have introduced the term **Fallacy.** This word is sometimes used to signify any false statement, erroneous belief, or mental confusion of any kind. This leaves the meaning of the word too vague. In the logical sense, a Fallacy is a violation of some rule or regulative principle of reasoning: in a word, it is a " mistake in reasoning."

(6) We may add a note on what are called **Correlative Names.** Two names are said to be " correlatives " or " correlative names " when the objects denoted by them are related through some common fact or principle: " father " and " child " through the fact of parentage; " king " and " subject " through one of the modes of government; and a name which has no correlation in this sense, has been called an " absolute " name. This distinction is of no logical importance; we have mentioned it only because some writers have given it a place in elementary logical doctrine.

It is, however, of great logical importance to understand that no object can be thought of entirely without relation to any other object; and in this sense (which is fundamental) *there are no non-relative or absolute names.* Everything is relative to other things, even on a superficial view; and if we imagine ourselves to be knowing or investigating its connections as completely as possible, " root and all, and all in all," its relations to other things would be found to have increased in extent the further our knowledge had penetrated. Hence every conception which we form is relative to

something else; whenever we think of a thing we are, at least, distinguishing it from other things. We think of a table, and the table is at once opposed at least to vacuity, if not to other articles of furniture. In this sense, every name is " relative."

3. Limits of Connotation

The logical ideal is that each term shall have a fixed and definite meaning: more precisely, that its connotation shall consist of a definite group of attributes which are sufficient, and neither more nor less than sufficient, to mark off the class (which it denotes) from all other classes. These attributes are expressed in the definition of the term.

This is a logical ideal rather than an actual characteristic of our thinking. For this reason some of the rules of Logic seem artificial—they are not intended to apply to the shifting connotations of many of our ordinary terms. Logically, it is our business to make the meanings of our terms definite, and to keep them so, changing them only when a real advance in knowledge requires it. Thus, in Plato's time, the connotation of the term " sun " was—" the brightest of the heavenly bodies which move round the earth." This clear and distinct idea had to be changed to what we now mean by " the sun " in consequence of advancing knowledge.

We cannot make the connotation of a term include *all the known qualities* common to the members of the class. With the growth of experience and knowledge we usually find that many of these known qualities are unessential, and some are insignificant from every point of view, and we simply leave them out of account in forming our idea; hence they do not form part of the connotation. In the case of the term " man," for instance, its logical connotation does not include the peculiar shape of the ears, or the capacity for laughter, or other known qualities common to

the class. It is sufficient if the connotation includes the *essential* attributes; and by the " essential " attributes we mean those attributes on account of which the name is given, and in the absence of which it would be denied.

In the next place we will compare the relation between change of denotation (*i.e.* increase or decrease of denotation) and change of connotation in the case of class names. The connotation of the term " ship " is definite enough to serve for an example. Increase the connotation to " steam-ship ": by this we reduce the denotation, since there are fewer " steam-ships " than " ships." Increase the connotation to " screw steam-ship ": then the denotation is still further decreased. In such cases, we may form a series of increasing connotation and decreasing denotation, thus: ship, steam-ship, screw steam-ship, iron screw steam-ship, British iron screw steam-ship. Hence the following rule has been given: " As connotation increases, denotation decreases; as denotation increases, connotation decreases." This rule is valid only under certain important limitations.

(1) The rule assumes that the connotations of the terms are definite and fixed.

(2) It applies only to names of classes which can be arranged in a series, in ascending or descending order of divisions and sub-divisions.

(3) It does not imply that there is a mathematically " inverse ratio " between such changes of connotation and denotation. There need not be any kind of proportion between successive increases of connotation and the corresponding decreases of denotation. Thus, from " civilised men " to " native of Ireland," the increase of connotation is not large; but the decrease of denotation is enormous. There is no such relation between connotation and denotation as can be formulated mathematically.

(4) It does not apply to a term whose meaning changes through increasing knowledge, or through increase in the number of individuals to which it applies. Increase of population does not change the meaning of the term "man."

(5) It applies only when the "increase of connotation" means the additional of a really new predicate. Then, to change "man" to "mortal man" makes no difference to the denotation, because the quality of mortality belongs to all men.

The best illustrations of the rule are found in the sciences of classification. The reader who has any effective knowledge of Botany or Zoology will have no difficulty in verifying this statement for himself. The ancient logicians were fond of the following example, which has therefore a certain historical interest:

CONNOTATION LEAST, DENOTATION HIGHEST

Being
(*i.e.* anything existing—"being" in the most general sense)

|

Material being
(*i.e. matter* in the widest sense)

|

Organic material beings
(*i.e.* the whole world of life animal and vegetable)

|

Sentient organic material beings
(*i.e.* animal life)

|

Rational sentient organic material beings
(*i.e* humanity)

|

This Man.

CONNOTATION HIGHEST, DENOTATION LEAST.

4. The Predicables

We have now to explain a classification of terms called by the name of "The Predicables." It has come down

from Aristotle through the later Greek and the Roman logicians, and is historically important. We now speak of "terms," because the classification applies only to names when standing in their places in logical propositions.

The Predicables are an attempt to classify the ways in which the predicate of an affirmative logical proposition may be related to its subject. We will illustrate by some simple examples, showing the way in which the ancient logicians arrived at these distinctions.

(1) Consider the following propositions:—

(*a*) "Man is *a rational animal.*"

(*b*) "A triangle is *a three-sided plane rectilineal figure.*"

(*c*) "A square is *a four-sided plane rectilineal figure with all its sides equal and one of its angles a right angle.*"

Each of these propositions states the essential attributes of the class denoted by its subject, *i.e.* those attributes on account of which the name is given, and failing which, it could not correctly be given. In propositions of this kind the predicate is a **Definition.**

(2) Consider the following:—

(*a*) "Man has *the power of speech*"; or "Man is *capable of progress in knowledge to an indefinite extent.*"

(*b*) "A triangle has *its three interior angles together equal to two right angles.*"

(*c*) "A square has *all four of its angles right angles.*"

In each of these propositions, the predicate is the name of a quality which is not part of the definition but which it follows from or is a consequence of the qualities named in

the definition. In propositions of this kind, the predicate is called a **Proprium** of the class named in the subject. It is better to use the Latin word *proprium* (plural *propria*). The English word "property," sometimes given as its equivalent, has a usage too wide for this purpose.

(3) Consider the following:—

(*a*) Men are *animals.*

(*b*) Triangles are *plane rectilineal figures.*

(*c*) Squares are *four-sided plane rectilineal figures.*

In each of these propositions the predicate is the name of a wider class in which the class named in the subject is included. We say "a wider class," because (*a*) there are other kinds of animals beside "men," (*b*) other plane rectilineal figures beside triangles (*i.e.* other figures having the characteristic of being bounded by straight lines all in the same plane), (*c*) other figures, beside squares, having the characteristic of being bounded by four straight lines all in the same plane. In propositions of this kind, the class named in the predicate is called the **Genus** (plural *genera*), of which the class named in the subject is a **species** (but not the only "species").

(4) Consider the following:—

(*a*) Man is *rational.*

(*b*) A triangle is *bounded by three straight lines.*

(*c*) A square has *its sides equal and one of its angles a right angle.*

In each of these propositions, the predicate is the name of a quality which differentiates or distinguishes the class named in the subject from other classes included (along with it) in the same *genus*, although the *genus* itself is not

named). In propositions of this kind the predicate is called a **Differentia** (plural, *differentiae*).

(5) Consider the following:—

(*a*) Some men *live for over a century.*

(*b*) Some triangles have *sides of such-and-such a length.*

(*c*) This square is *drawn on a given base line.*

In each of these propositions, the predicate is the name of a quality which may or may not belong to the class named in the subject, or does not *necessarily* belong to it, or is not *essential* to it. In propositions of this kind, the predicate is called an **Accidens** of the subject. The Latin word *accidens* (plural *accidentia*) is preferable to the English word "accident," which has too wide a usage. The fact that an *accidens* is unessential may be recognised in two ways: it may belong to some members of the class and not to others—"this clover has four leaves"; or it may belong to an individual at one time and not at another—"Socrates is standing in the Agora."

Some writers on the subject have introduced the distinction of "separable" and "inseparable" *accidentia.* Thus, the clothes in which a man is dressed form a separable *accidens,* because they can be changed, as can also his position, and many other circumstances; but his birthplace, the colour of his eyes, etc., are inseparable *accidentia,* because they can never be changed, although they have no necessary or important relation to his character. "The Ethiopian cannot change his skin, nor the leopard his spots."

It will be found that every proposition must come under one or other of these five heads, and that most of the assertions which we make in ordinary life are cases of "accidental" predication.

Now consider the examples in (3) and (4) above. Combining the propositions (3, *a*) and (4, *a*), we get proposition (1, *a*)—"man is a rational animal," or, in strict logical form, "All men are rational animals." In other words, the name of the *genus*, together with the name of the *differentia*, gives the *definition*; in this case, "animal" is the *genus*, "rational" the *differentia*.

The genus, therefore, is a wider class including within it sub-classes or **Species.** The genus is narrower in defined qualities (in connotation) than the species, but larger or wider in denotation; while the species is richer in defined qualities but narrower in denotation. The species must contain all the qualities of the genus, as well as a certain additional quality or qualities by which the several species are distinguished from each other. These additional qualities form the *differentia*—the quality or group of qualities which mark out one species of the *genus* from any other species included in it. An example may be given in tabular form. Consider the genus "plane rectilineal figure":—

Plane rectilineal Figures

Three-sided (Triangles)	Four-sided (Quadrilaterals)	With more than four sides (Polygons)

Triangles have all the properties of plane rectilineal figures, and are "three-sided" as well. Quadrilaterals have all the properties of plane rectilineal figures, and are "four-sided" as well. We are thus able to define the species "triangle" by adding the *differentia* "three-sided" to the genus rectilineal figure—"a triangle is a three-sided rectilineal figure."

The following is a *summary statement* of the distinctions which we have explained above:—

(*a*) A *Definition* is a proposition which states the essential attributes of the class denoted by its subject; *i.e.* those attributes on account of which the name (standing as the subject-term of the proposition) is given; and, failing which, it could not correctly be given.

(*b*) A *Proprium* is an attribute which is not part of the definition but which follows from or is a consequence of the attributes named in the definition.

(*c*) An *Accidens* is an attribute which not only does not form part of the definition of a class but is not necessarily connected with any attribute included in the definition.

(*d*) A *Genus* is a wider class made up of narrower classes called *Species*.

(*e*) A *Differentia* is an attribute or attributes by which one species is distinguished from all others contained under the same genus.

To these statements we must add that the Definition of a class name states the next higher Genus to which the class belongs together with the Differentia which distinguishes it from that Genus.

The terms "genus" and "species" are strictly correlative (see page 29). A genus implies species, and *vice versa*; and both names derive their meaning from the same principle or fact. The relation of species to genus is that of "subordination"; and the relation of different species (of one and the same genus) to each other is that of "co-ordination." Thus in the last example given, triangles, quadrilaterals, etc., are *co-ordinate* species of the genus "plane rectilineal figures."

In Logic, any pair of classes of which one is subordinate to the other is related as species and genus. But in Natural History these terms are given a particular place within a hierarchy of divisions and sub-divisions: "Kingdom," "Group," "Class," "Order," "Family," "Genus," "Species," "Sub-species" (if necessary), or "Variety." Logically, each of these is genus to the one which follows it.

It is theoretically possible to arrive at a genus which has no class above it, and hence is called a *summum genus*; and, at the other extreme, to arrive at a species which cannot be further sub-divided except into individuals, and is therefore called *infima species*. An example has already been given (see page 32), where the *summum genus* is "being (in general)," and the *infima species* is the class "man."

We have spoken of "adding" the *differentia* to the genus. The expression is in general use. But if understood mathematically it is misleading. In fact, we must be on our guard against the use of mathematical terms in connection with connotation. Attributes or qualities may be distinguished and (to a certain extent) defined; but they are not separate units like individuals in a class. Through overlooking this fact, the relation between change (increase or decrease) of connotation and decrease or increase of denotation was formerly supposed to be one of "inverse ratio."

CHAPTER III

DEFINITION, DIVISION, AND CLASSIFICATION

1. Definition

We have already referred to the logical meaning and purpose of Definition. We may accept the statement that "Definition is the analysis of the connotation of a term": but the statement requires careful explanation.

If a term is to be definable, we must assume that it has a fixed and definite connotative meaning, consisting of those attributes which are sufficient, and neither more nor less than sufficient, to mark off the class which it denotes from other classes; or, to express the same thing in different words, we assume that the connotation of the term consists of those qualities on account of which we may apply it as a name of a given class of objects, and in the absence of which we could not correctly apply it to that class of objects. For Logic, therefore, the question of Definition is the question of the best way of expressing or formulating the connotative meaning of a term, which we assume to be already known or discovered. All definitions define the meanings of terms or names.

We may say, therefore, that definition is concerned with words, but not with *words only*. Indeed to speak of "words only," in this connection, would be nonsense. The phrase could only be equivalent to "words apart from any meanings which they would have"; and a "word" apart from any meaning which it could have would be a mere *sound or noise* when heard and a mere *shape* when seen. We may add that mental association of another noise or shape with it would not constitute a "meaning," either in the logical or in the psychological sense. On the development of "meaning," see Stout and Mace, *Manual of Psychology*, fourth edition, 1929, Book I., Chapter 4 and Book III., Chapter 1; and McDougall, *Outline of Psychology*, second edition, pages 250 ff.

With regard to the formulation of our definitions, the most important practical rule is to define *per genus et differentiam.* This has been illustrated when we were explaining the terms genus, species, and *differentia*. (See Chapter II., page 34.) It means that we distinguish the object from the class which it most resembles. We start with the class in whose connotation the greater part of the features which we wish to indicate is already contained; thus, if we wish to define the "phoenix," we begin by saying, "it is a bird." We refer it at once to a genus which is assumed to be familiar to other minds. The nearest or "proximate genus" is chosen because then a simpler specific difference is sufficient to distinguish the object from that genus. The "proximate" genus is simply the next higher genus. Otherwise the *differentia* would be imperfect, and some necessary attributes omitted. A child might be tempted to define the species "chair" by falling back on the genus "wooden thing."

It is, however, of educational interest to find that children learn to use the logical rule for definition with reasonably good results. An experienced Inspector of Secondary Schools has given some examples from among pupils in a Second Form. "They were asked to define the name 'hat'; a few questions from the teacher elicited the genus 'article of dress,' and the *differentia* 'for wearing on the head.' Thus the definition given was, 'a hat is an article of dress for wearing upon the head.' A pupil of nine in the same class defined a 'chair' as 'an article of furniture used for sitting on.' For such a young child, the definition was good, but the *differentia* 'for sitting on' is obviously too vague, for the same definition would apply to other kinds of seats. The definition of 'book' proved more difficult, and the teacher occupied a considerable time before she could obtain a suitable genus. When at

last the genus ' a number of printed sheets of paper of the same size' was suggested, the teacher wisely refrained from quibbling over the suitability of the term ' sheet ' or of the words ' the same size,' and accepted the suggestion. The *differentia* finally decided upon was ' bound together,' though one child objected to this on the ground that ' bound together ' might mean bound at all four edges . . . The whole discussion showed how even young children can be induced to enter into the real significance of words."[1]

We must observe that the definition of the names of common and familiar objects presents more difficulty than the definition of *technical terms* in the various Sciences and Arts. We use the names of familiar things without any clear idea of their connotation. We are content to know their denotation, at any rate sufficiently for the practical purposes of life. Indeed, throughout ordinary thought and common life, people are much more sure of the particular objects to which a name applies than they are of the qualities in the objects on account of which the name is given. Thus, when we speak of such a thing as " an oak tree " or " a rose "—or " a beautiful object," " a good action "—it is more easy to bring forward actual instances of these things than to " analyse the connotation " of the name which we use.[2]

It has been said that " to obtain perfect definitions is practically impossible." This is an exaggeration. Its truth depends upon the assumption which we have already stated, and which must be granted if terms are to be definable: that the connotative meanings of terms are known and are definite and (at least for the time being) are fixed. This assumption is realised only partially and

[1] F. W. Westaway, *Scientific Method,* fourth edition, pages 21-2.
[2] See Bertrand Russell, *Introduction to Mathematical Philosophy,* second edition, page 12.

imperfectly in the case of most general terms in current use; but it does not follow that these terms are logically *indefinable.*

A term may be " indefinable " for either of two reasons: we may be unable to assign it to any genus, or we may be unable to state precisely any *differentia* distinguishing it from the genus. Consider the case of Singular terms (proper names and uniquely descriptive names). A Singular term is the name of one unique object (person, place, or thing), to the exclusion of every other. Such a term is logically indefinable, because the countless peculiarities which constitute its individuality cannot be enumerated *per genus et differentiam* or in any other way. We can usually assign a Singular term to a genus; but we cannot definitely state a complete *differentia.* " Cambridge is a university town "; " Socrates is a man ": each of these statements names a genus, but it is evident that neither is a definition. A Singular term, as the name of a unique individual, is said to be *sui generis*, of its own class, since it cannot be referred to any genus as a species of it.

Singular terms, however, are not the only names which are logically indefinable. The names of simple qualities, for instance, the various elementary sensations, such as " hot," " red," and the names of elementary mental qualities and activities, such as " pleasure," " pain," " attention," cannot be defined by an enumeration of their attributes; they are too simple. When we can state a genus for such a term, we find that we cannot state a *differentia.* And names of the most general or universal qualities of material bodies, such as " time," " space," are indefinable for a like reason. The name of the most general quality of mind, the quality of " consciousness " is, indefinable in the same way. The following may be given as a description or indication of what " consciousness " is,

but it is not a definition: "What we are when we are awake, as contrasted with what we are when we fall into a profound and dreamless sleep, that it is to be conscious; what we are less and less, as we sink gradually down into a dreamless sleep, or as we swoon slowly away; what we are more and more, as the noise of the crowd outside tardily arouses us from a 'nap,' or as we come out of the midnight of the typhoid-fever crisis—that it is to be conscious."[1]

2. Rules for Stating Definitions

There are a number of *rules for the formulation or statement* of a definition, which have come down to us from the ancient logicians. They are rules for the definition of any general or class name whose connotation is supposed to be known.

(1) The fundamental rule is that the definition must explicitly state the essential features of the objects to which the name is applicable. The essential qualities, we repeat, are those on account of which the name is given to the objects in question, and in the absence of which it could not correctly be given. And these are the qualities from which the largest number of others *are found to flow as consequences.*

(2) The term expressing the definition must be simply convertible with the term defined. In other words, when the definition is expressed as a proposition in strict logical form, as for example, in the case of the proposition "All triangles are plane rectilineal three-sided figures," we may simply (or without other alteration) convert the proposition, and say, "All plane rectilineal three-sided figures are

[1] G. T. Ladd, *Psychology, Descriptive and Explanatory*, page 30.

triangles." This rule secures that the definition shall be neither too wide nor too narrow. In the following examples the definition fails by being *too wide.* "Tin is a metal lighter than gold" (other metals than "tin" are lighter than gold). "Man is a vertebrate animal" (there are many kinds of vertebrate animals beside "man"). "Eloquence is the power of influencing the feelings by speech or writing" (many things, said or written, influence the feelings, but are not "eloquent"). In the following examples, on the other hand, the definition fails by being *too narrow* (an error usually arising from the attempt to define a higher class by a lower which is included in it): "Wealth consists of money." "Wealth consists of natural products." The student of Economics will recognise these errors, each of which consists of a fatally narrow definition. "Justice is minding one's own business." Even if we put a large interpretation on the term "business," and understand "minding" in a moral sense, the definition is still too narrow.

(3) The definition should not be obscure. This arises usually from the use of expressions which are less familiar than the one to be defined, thus defining "the obscure by the more obscure" (*obscurum per obscurius*). Herbert Spencer's formula for the process of Evolution, if given as a definition and not as the summary of a treatise, is a striking example of *obscurum per obscurius:* "Evolution is to be defined as a continuous change from indefinite incoherent homogeneity to definite coherent heterogeneity of structure and function, through successive differentiations and integrations." The defect of "obscurity" in definition may also arise from the use of *eccentric* or *metaphorical* expressions. "Thunderbolts are the winged messengers of the gods." "The body is the visible

garment of the soul." "Prudence is the ballast of the moral vessel." Such statements, though useless as *definitions*, may be highly suggestive as *metaphors*. We must add that scientific definitions, expressed in the technical language of a particular science—Chemistry, for instance—are not examples of the fault referred to above, provided that the technical terms used in the definition have themselves been defined.

(4) A definition must not use the term to be defined. An apparent definition which commits this fault is said to be "circular" or "tautological" and was formerly described as a *circulus in definiendo*. An obviously "circular" definition may be intended to be an epigram: thus, "an archdeacon is one who exercises archidiaconal functions" might be a way of saying that "nobody knows exactly what the duties are." But circular definitions are often made through the use of synonymous or equivalent words or phrases: "Chance is the cause of *fortuitous* events." "Force (in physical science) is a *power* which tends to produce motion." "Justice is giving to every man *his due*." (There are other points of difficulty in the first and second of these definitions: we are not entitled to make "chance" a species of the genus "cause"; and if the meaning of "force" is obscure, the meaning of "tends" is still more obscure.) The use of *correlative* terms sometimes leads to a circular definition: "A cause is that which produces *an effect*." Mere repetition of a word does not vitiate a definition: we may define "an equilateral triangle" as "*a triangle* with three equal sides," having already defined "triangle" and being now concerned to define "equilateral."

A fifth rule is sometimes added, to the effect that the definition should not be negative where it can be positive.

Using the terminology of Chapter II., page 26, we may express this by saying that a term cannot be defined by its *contrary* or by its *contradictory*.

Sometimes a Definition in negative form, stating what the Subject is not, may be appropriate when we have to define a negative term. For example, the name " alien " has as its *differentia* the negative attribute, " one who is not a citizen of the British Commonwealth." But usually the " negative definition " should be avoided.

The following is a *summary statement* of the Rules of Definition:—

(1) The definition must state the essential attributes of the class denoted by the Name defined.

(2) It must not be too wide or too narrow.

(3) It must not be expressed in obscure, figurative, or ambiguous language.

(4) It must not use the Name to be defined or any synonym of it.

(5) It must not be negative when it can be affirmative.

Hitherto we have been speaking of the method of logically formulating the definition of a name whose connotation is supposed to be known. The question is regarded from another point of view, and the word " definition " is used in another sense, when we consider the method by which definitions are discovered or arrived at. The process of *arriving at* definitions is just the process of science as a whole. In this sense it has been said that " the business of definition is part of the business of discovery "; that " discovery and definition go hand in hand "; that " definition is the end and aim of science." In this sense, we may go further and say that definition is discovery, and discovery is definition. The only reason for retaining the word " definition " *in this connection* is that it serves as a name for the summing up of our discoveries of what things really are. And though it is true that all our definitions are of terms or names and aim primarily at fixing the meanings of names, it is also true that they have a more or less direct reference to reality. The connotations of our terms depend on the general state of knowledge to which we have attained, and often on the

general state of civilisation; and this dependence may be made a matter of deliberate investigation.

We begin by thinking of an object in a loose general way as a whole made up of parts which are familiar. Such an idea may be little more than a mental picture; but as long as it is precise enough to avoid confusion with other things, we are content. But reason suggests a step in advance: to ascertain the characteristics which the object has in common with other species of its genus, and to distinguish it from those species. Even in the method of *pointing*, of showing the denotation in the absence of any serviceable definition, there is a stimulus to mental comparison in order to distinguish the common element which we wish to get at, and make it into a definition. Reference to instances is essential to the discovery of definitions, and this reference will imply, or actually take the form of, an arrangement into classes according to resemblances and differences. Then, as we shall see when treating of the Logic of Induction, we cross the frontier into the territory of the Sciences, where "discovery is definition." This constitutes the most important difference between *mathematical* and *physical* science. In mathematics, our definitions are not matters to be discovered or ideals to be reached; they are principles with which we start.

For a brief statement of the Aristotelian view of Definition, see Mellone, *Introductory Textbook of Logic*, Chapter V., §§ 4-6, and for a full exposition, Joseph, *Logic*, second edition, Chapter IV. For recent developments of the doctrine of Definition, see Stebbing, *A Modern Introduction to Logic*, Chapter XXII. (where many further references are given).

Some logicians assume that, in addition to definitions *per genus et differentiam*, there are what they call other "kinds" or "types" of definition. These would be more accurately described as different meanings which have been given to the word "definition"; and for some of these meanings, some other name would be more appropriate. For our present purpose, three of these "kinds of definition" may be explained.

(1) Closely akin to strictly logical definition, is the type of definition which results from legal enactments. In Acts of Parliament, for instance, an ordinary term, such as "person," "owner," "parent," "parish," "factory," has a special and precise meaning given to it—this being

artificially made, and constituting a "conventional connotation" not capable of growth by advance of knowledge, as in the case of scientific terms.

(2) What are called "genetic" definitions aim at showing us indirectly how to form an idea of the object: "A sphere is a solid figure formed by the revolution of a semicircle about its diameter, which remains fixed." Theoretically and ideally, a *genetic* definition would show us the way in which the object has actually been produced in Nature: and then the "definition" would become a causal explanation.

(3) Finally, in the so-called "Definition by Type," an individual member of a class is taken as representing the class. This method, which is not really definition at all, may vary greatly in definiteness and value. At one extreme, it is exemplified in merely pointing to a member of the class in question and saying, "I mean something like that." At the other extreme, we have the "enumerative descriptions," as we may call them, seen in the naturalist's list of marks for identifying a plant or an animal, or in any matter-of-fact description which reads its objects piecemeal, seeking to reach the idea of the whole by "traversing hither and thither and putting together the contents of the field." If we use the terminology explained above, we may say that in *description* of this kind, inseparable *accidentia* are often used, with or without some of the *propria*, so as to enable us to recognise the objects denoted by the name.

3. Division

What is technically termed "logical Division" is closely allied to definition. The former has reference to denotation, the latter to connotation. Just as definition

is the analysis of the connotation of a term (the connotation being assumed to be known), so division is the analysis of the denotation, *i.e.* of the genus to which it belongs; this genus being assumed to be given. We are not forming or making a class, but dividing one. The "Rules of Division," therefore, are only *an expanded statement of the relation of a genus to the species which compose it.* These Rules may be reduced to three.

(1) In dividing a genus, the basis of division or *fundamentum divisionis* must be a quality common to the whole extent of the genus, and the species must be distinguished according to the different modifications of this quality which they possess; so that the *differentia* of each species will be some definite modification of the *fundamentum divisionis.* For example, if it is proposed to divide the genus "plane rectilineal figures" on the basis of "number of sides," we should get a result which may be thus represented:—

Plane rectilineal Figures

Triangles (three-sided)	Quadrilaterals (four-sided)	Pentagons (five-sided)	Hexagons (six-sided)	Figures with more than six sides

(2) Each act of division must have one basis only: this secures that the species (as in the example just given) shall be mutually exclusive. Violation of this rule leads to what is called "cross division," which practically means that the species overlap. A gross example would be the division of "Churches" into "Gothic, Episcopal, and Evangelical," where there are three bases of division, architecture, government (or organisation), and dogma; and no account is taken of the many different kinds of each; or the division of "metals" into "white, heavy, and

precious," where again there are three bases of division—colour, weight, and commercial or aesthetic value.

(3) The constituent species must be together equal to the genus; in other words, the division must be exhaustive. It must not "make a leap," *i.e.* leave gaps (*divisio not facit saltum*). Illustrations are scarcely needed. Thus, the division of "Governments" into "republican and despotic," of "men" into "saints and sinners" (*i.e.* those who are wholly saints and those who are wholly sinners), or "religions" into "the entirely true and the entirely false," all alike fail to be exhaustive.

Logical division is of course essentially different from physical partition or any kind of analysis of a whole into its parts or aspects. Thus, the "division" of Ireland into "Northern Ireland" and the "Irish Free State" implies a territorial partition and a difference of government; but it is not the division of a genus into co-ordinate species. And the "division" of mind into "thought, feeling, and will," does not mean that "thought," "feeling," and "will" are three species of the genus "mind"; it is merely a rough-and-ready analysis of mental life into three co-operating factors. In like manner the "division" of body into "extension, resistance, weight, etc.," is a distinction of aspects of every material body, not a division of co-ordinate species of the genus "matter."

4. Dichotomy

It was formerly supposed that to obtain a strictly logical division, every class must be divided into two sub-classes, distinguished by the possession or non-possession of a single specified difference. Each of these sub-classes must be again divided according to the possession or

non-possession of any relevant or possible quality which may be selected. For example:—

Parts of Speech

nouns — not nouns

pronouns — not pronouns

verbs — not verbs; etc.

Such an arrangement is not "logically perfect" or "strictly logical" in any proper meaning of these words. It is no more independent of our knowledge of the facts than any other kind of classification; and the negative term in each case is a *material contradictory* (see Chapter II., page 27), the extent of which in other respects is entirely indefinite. The dependence of the process on material knowledge is effectively indicated in the following case:—

Vertebrates

with lungs — without lungs (*fish*)

with mammae (*mammals*) — without mammae

with wings (*birds*) — without wings (*reptiles*)

We could not construct such an arrangement without material knowledge of "mammals," "birds," "fish," and "reptiles,"—without knowing, for instance, that fish are not mammals and have no wings, that no mammals have

wings, and that the "wings" of "flying fish" and of "bats" are not physiologically true wings as in the case of birds.[1] Aristotle discussed this method of division, which he called διχοτομία, *dichotomia*, from which the modern technical term "Dichotomy" is derived. He was aware of its dependence upon material knowledge. Nevertheless the ancient logicians attached much importance to it. They were fond of the following special illustration of it:—

Substance

corporeal *i.e* body — not corporeal

organic *i.e.* living being — not organic

sentient *i.e.* animal — not sentient

rational *i.e.* man — not rational

Socrates — Plato — and others

The most serious objection to Dichotomy from the scientific point of view is, that if we *know* the sub-classes and divisions included under the *negative* term, then there is no necessity to indicate them by any kind of merely contradictory name; and if we do not know them, then the negative term is not the name of a *class* at all, and we have not a logical division. The only use of such a method

[1] For the last two examples I am indebted to Westaway, *Scientific Method*, fourth edition, Chapter XVIII., § 4.

is in occasionally helping us to mark distinctions, as a preliminary to a genuine classification; thus we may find it useful to divide organic beings into sentient and non-sentient, flowers into scented and scentless, fluids into coloured and colourless, and so on. Sometimes, again, when we are arranging objects, as books in a subject-catalogue, and further division becomes impossible, we add a class "miscellaneous," which really means "All those *not* in any named class." But we never form a *class* that can be indicated by a mere contradictory term.

5. Classification

All general names imply the existence of classes corresponding to them. We have already illustrated this fact (see Chapter II., page 19) in the case of class names. Classification is the arrangement of objects according to their likenesses and differences. We may say that this is the beginning of knowledge. Any complete act of sense-perception is an example of it. To *perceive* an object is to assimilate or classify it—"This is so-and-so." And what we find ourselves doing, in attempting to understand any object of our experience, is first to find a class for it, and then to compare representative individuals of the class with it, taking into account also the contrasted classes. If we cannot find a class—in other words, if the object is like nothing in our previous experience, we are completely at a loss. In the sciences, classification is the systematic arrangement of animals, of plants, of minerals, etc., according to important likenesses and differences, for the sake of studying their form, structure, and function. We have now to see what this implies.

We must first understand the difference between a "natural" and an "artificial" classification. The general

rule for forming a **Natural Classification** may be given thus: *Place together in classes the things that possess in common the greatest number of attributes.* But this does not mean that classification depends on the counting of attributes. As we have already observed, the qualities of objects are not units capable of arithmetical enumeration. The result defined in the rule may be obtained in ways other than counting. (1) When we class together human beings of different races as having in common the attributes of manhood or humanity, we have taken as our basis an *essential* quality—that is, a quality from which the largest number of others are found to flow as consequences, or which has a determining effect on the largest number of others, or (to put it more generally) the quality which carries with it the largest number of other qualities. (2) Sometimes the required result is secured by taking as a basis of classification a quality which is an *index or accompaniment* of a larger number of others, although these latter may not be in any important respect affected by it. Such a quality is an " inseparable *accidens*." Thus, the squirrel, the rat, and the beaver are classed together as " rodents," the difference between their teeth and the teeth of other *mammalia* being the basis of division, because the difference in the teeth is accompanied by differences in many other qualities.

In either of these cases, the basis of classification consists of numerous common qualities taken together, and we have a natural classification as defined above. It is evident that such a classification enables us to make the greatest number of general assertions about the class, and to infer about any other member a great part of what we know about any one; because its members have the greatest number of points of mutual resemblance, and the fewest points of resemblance to members of other groups. Such

is the technical meaning of " natural " in this connection. But its ordinary meaning is also relevant here; because the classifications which are " natural " in the technical sense are those which correspond to the groups or divisions which occur in nature. And the systems of classification worked out in such sciences as zoology, botany, crystallography, or mineralogy, all aim at setting forth the real structure and natural affinities typical of each class.

On the other hand, there are classifications which are made on the basis of a single attribute, or a very few attributes. These are called " artificial " partly in contrast to the " natural " classifications, partly because they are constructed to serve all manner of purposes other than scientific or theoretical. An **Artificial Classification** is usually made on the basis of one attribute only. Examples are easily found; the arrangement of words in a dictionary, the object being to find any word as easily as possible; the arrangement of books in a library, according to size, for economy of space; or according to the initial letters of the authors' names; or according to the language in which they are written.

Scientific classification is not invalidated by the fact that in Nature there are no definite frontier lines dividing one group of facts from another—as, for example, between the animal and the vegetable kingdoms. The " frontiers " may be compared to *regions* rather than to *lines*. But it is no valid objection to a classification that cases may be found which fall on or in one of these marginal regions. This is inevitable in the nature of things. None the less, the classification is a good one if in those instances which do *not* fall in the marginal region, the distinctions marked by the classification are such as it is important to mark, such that the recognition of them will help the inquirer towards the desired goal.

Formerly it was customary to treat "Division" under the head of "Deductive Logic," and "Classification" under the head of "Inductive" Logic. But there is no reason for the separation. In Division, we start with a *given* class and divide it into sub-classes. In Classification, we start with individual objects, or comparatively small groups, and arrange them systematically as we have shown above. Division works downwards from the general to the special; while Classification works upwards from particulars through the special to the general.

CHAPTER IV

CLASSIFICATION OF PROPOSITIONS

Propositions may be classified in different ways; and no classification can include all possible forms. The classification which we now proceed to explain may be called " traditional," because it goes back to the Greek logicians and ultimately to Aristotle; although in the course of its history it has assumed forms for which Aristotle is not responsible.

1. The Traditional Classification

We have seen that a proposition is an assertive sentence, a statement which admits of being either true or false. Three kinds of propositions are distinguished: (1) Categorical, (2) Hypothetical, (3) Disjunctive.

(1) A **categorical proposition** is one in which no statement is made of any condition or limitation under which the proposition is true. A " categorical " proposition, in the strictly logical sense of the word " proposition," must be expressed in the form " S is P," with a distinct Subject (" S "), Predicate (" P "), and Copula (" is "). The Subject is that about which the statement is made; the Predicate, that which is stated about it; the Copula, in the form of the present tense of the verb " to be," expresses the mental act of assertion—the fact that I think of S and P as really joined together in the way in which the proposition states them to be.

(2) A **hypothetical** or **conditional proposition** predicates P of S under a condition, that is, provided certain circumstances are supposed or granted: " If metals are heated,

they expand," where the condition is that the metal is heated; or, again, "If money is scarce, prices are low," where the condition is that an insufficient quantity of the standard coinage is in circulation. The symbolic forms of hypothetical propositions are: "If A is B, it is C," as in the first example; and "If A is B, C is D," as in the second.

(3) A **disjunctive proposition** is one which makes an alternative predication: "All candidates must be graduates either of Cambridge or of Oxford"; or, again, "Either the witness is perjured, or the prisoner is guilty." The symbolic forms are, as in the two examples, "A is either B or C," and "Either A is B or C is D." There may, of course, be more than two alternatives. These also may be called "conditional" propositions; but the condition is "either . . . or" and not "if . . . then." We shall return to the consideration of hypothetical and disjunctive propositions after we have explained the different forms of the categorical proposition.

When we make a categorical assertion we must either (*a*) affirm something of the subject we speak of, or (*b*) deny something of it. This distinction, of affirmation and negative propositions, is called one of **Quality**. The reader will observe that this is a purely technical use of the word "quality," in connection with the affirmative or negative character of logical propositions. Again: the affirmation or denial may be made (*a*) of a whole class or kind of things, or (*b*) of part of such a class, or (*c*) of some one particular thing. This distinction, of the whole or part of the class referred to, is called one of **Quantity**.

According to Quality, then, propositions are either *affirmative* or *negative*. The affirmative expresses a union between subject and predicate in the sense that the

attributes signified by the predicate belong to the subject; thus in the proposition "Fixed stars are self-luminous," the quality of shining by their own light is said to belong to the heavenly bodies called fixed stars. The negative expresses a separation of subject and predicate in the sense that the attributes signified by the predicate do not belong to the subject; "gold is not easily fusible" states the quality of being easily fusible does not belong to gold. The typical logical forms of affirmation and denial are, respectively, "S is P," "S is not P." The reader must bear in mind that in this formal expression of the negative proposition, the word "not" belongs to the copula:—

Subject	*Copula*	*Predicate*
S	is not	P

This is of importance because propositions may occur in which a contradictory term in the form "not P" stands as predicate, so that the word "not" belongs entirely to the predicate in the formal expression of the statement:—

Subject	*Copula*	*Predicate*
	is	not-P

For the future we shall form contradictory terms by the use of "non-" with hyphen—the Latin word for "not"; for example: "non-A," "non-human." We shall thus distinguish the negative implied in these terms from the negative which belongs to the copula.

In the next place, we come to distinctions of Quantity. These apply both to affirmative and to negative propositions.

(*a*) The affirmation or denial may be made of every thing of a certain kind or class. In this case we have a

universal proposition, so called because the predicate is affirmed or denied of every instance of the subject—thus, in "All planets shine by reflected light," this quality is affirmed of each of the class of "planets"; and in "No men are utterly bad," this quality is denied of each one of the class "human beings." A proposition which is not already in the form "All S is P" or "No S is P," if it is really universal, can be expressed in this form without changing its meaning.

(*b*) The affirmation or denial may be made of part of a certain class. In this case the proposition is said to be **particular.** Its logical form is "Some S is P" or "Some S is not P"; "Some men are born great"; "Some statesmen are not practical." The particular proposition *in ordinary speech* is usually an assertion about some quantity between the two extremes of universal affirmation and universal denial, in reference to a class; in other words, it usually means "some only," "only a part." But we need some sign of quantity which would enable us to assert that "some S is P" while leaving it an open question whether *all* are, and to assert that "some S is not P" while leaving it an open question whether *none* are. Now in the logically "particular" proposition this need is met by the understanding that "some" is given no more than its minimum interpretation; in other words, it means "*at least* some." The only possible ground for taking "some" in the sense of "*only* some," in a logical proposition, is our knowledge of the subject-matter, not anything in the formal expression of the proposition; thus, in "Some men are black-skinned," we know as a matter of fact that the statement applies only to some groups of mankind; but we do not know this from the logical form of the proposition.

(*c*) The affirmation or denial may be made of some one object or individual, to the exclusion of every other. In this case the subject of the proposition is a Singular term (see Chapter II., page 22). The following are examples—not all, of course, expressed in strict logical form: "*I* am what I am"; "*He* has blundered"; "*Job* must have committed some secret sin"; "*This statesman* is not dishonest." In the traditional classification of propositions, which we are now explaining, any proposition whose subject is a Singular term is ranked as a universal, on the ground that the predicate refers to "the whole of the subject." It is unnatural to treat an individual as a class, but such is the traditional method. The paradox is to some extent overcome if we think of a Singular term as the name of a unique class with only one member.

On the whole, therefore, we have four possible forms of the proposition:—

Universal	affirmative	All S is P
	negative	No S is P
Particular	affirmative	Some S is P
	negative	Some S is not P.

The form "All S is P" is denoted by the letter **A**; "No S is P" by **E**; "Some S is P" by **I**; and "Some S is not P" by **O**. The propositions are often abbreviated thus: S *a* P, S *e* P, S *i* P, S *o* P. The letters were chosen because **A** and **I** are the first two vowels of *affirmo*, I affirm, and **E** and **O** are the vowels of *nego*, I deny.

The following simple examples of the four traditional

"standard forms," as they may be called, are illustrative of what we have said above:—

A = "All men are fallible";
E = "No men are perfect";
I = "Some men are wise";
O = "Some men are not learned."

This classification of categorical propositions still holds the field in the elementary treatment of logic; and owing to its historical importance, its meaning and use must be thoroughly grasped.

The reader will observe that in our discussion of the four standard forms we have understood the subject of the proposition as the name of a *class*, and the predicate as the name of certain attributes which are affirmed or denied of some or all of the class. But another interpretation, which is sometimes more convenient, is possible; namely, to understand both subject-term and predicate term as the names of classes. In that case, the propositions given above are interpreted as follows:—

A = "All of the class 'men' are included in the class 'fallible beings'";
E = "None of the class 'men' are included in the class 'perfect beings'";
I = "Some of the class 'men' are included in the class 'wise beings'";
O = "Some of the class 'men' are not included in the class 'learned beings.'"

It is not necessary to use this actual form of words in the propositions; but the possibility of this interpretation must always be borne in mind.

An additional observation of much importance must be made in reference to the copula. The logical use of the present tense of the verb "to be" in the copula has no reference to any particular *time*. It simply signifies that this affirmation or denial *is made*. If reference to some particular time is part of the proposition, it has to go into the subject, or (more usually) into the predicate; thus, "Gladstone was a wise Prime Minister" becomes, in logical form, "Gladstone is one who was a wise Prime Minister."

It is evident from mere inspection of the four-fold propositional scheme that the *extent* of the group referred to in the subject or predicate may be total or partial. We must now define these possibilities more precisely. This may be done most conveniently by understanding both subject and predicate as names of classes or groups. When a term—whether it is subject or predicate in a logical proposition—refers to the whole extent of the group which it names, that is, when we know from the form of the proposition that it so refers, then the term is said to be **distributed** in that proposition. And when a term does not explicitly refer to the whole of the group which it names, it is said to be **undistributed** in the proposition. What terms, then, are known to be distributed in the four propositional forms?

(1) Take first the *universal affirmative proposition*, "All S is P." It states that the whole class denoted by the subject is included in the class denoted by the predicate. The subject is obviously taken in its whole extent, as is shown by the word "all." The subject is therefore *distributed*. But what of the predicate? The proposition does not tell us whether the predicate-class is or is not wider than the subject-class. Either possibility may occur. Thus (*a*) "All metals are elements" means, in the

"class" interpretation, that the class "metals" is included in the wider class "elements"; and (*b*) "all equilateral triangles are equiangular" means that the class "equilateral triangles" is in the class "equiangular triangles," but here we know from the matter of the proposition, not from its form, that the class "equilateral triangles" is identical with the class "equiangular triangles." But we do not know from the form of the proposition whether the predicate is taken in its whole extent, as in (*b*), or only in part of it, as in (*a*). The predicate therefore is not distributed—*undistributed* being the word usually employed.

(2) The *universal negative proposition*, "No S is P," states the class denoted by the subject is altogether outside the class denoted by the predicate. The subject is taken in its whole extent and is therefore *distributed*. But if the whole of S is outside P, then the whole of P must be outside S. In other words, P also is taken in its whole extent and is therefore *distributed*.

(3) The *particular affirmative proposition*, "Some S is P," states that some at least of the class denoted by the subject is included in the class denoted by the predicate. The subject therefore is *undistributed*. What of the predicate? There are two possibilities; and the form of the proposition does not tell us which is to be taken. Then (*a*) "Some metals are brittle" means that part of the class "metals" is included in the class "brittle things," and we know that there are other brittle things than "metals"; and (*b*) "Some Europeans are Frenchmen" means that part of the class "Europeans" *coincides* with the class "Frenchmen." But the form of the proposition does not tell us whether the predicate is taken in its whole extent, as in (*b*), or only in part, as in (*a*). The predicate, therefore, is also *undistributed*.

(4) The *particular negative* proposition, " Some S is not P," states that some at least of the class denoted by S is outside the class denoted by P. Here, again, the subject is *undistributed*; but the proposition tells us, by its mere form, that the " Some S," whatever its extent may be, is wholly excluded from P; it falls entirely outside P and P falls entirely outside it. Hence the predicate is *distributed*.

Our results may thus be tabulated:—

Proposition.	*Subject.*	*Predicate.*
A	distributed	undistributed
E	distributed	distributed
I	undistributed	undistributed
O	undistributed	distributed

2. Expression in Logical Form

The exercise of paraphrasing ordinary or poetical or rhetorical assertions so as to express them in one or other of the four standard forms, is a valuable exercise in accuracy of thought and clearness of expression, and strengthens the habit of exact interpretation.

We spoke of " paraphrasing ": this means that Logic requires to be allowed to state explicitly all that is implicitly contained in the thought which the given assertion expresses. To do this, we ask first, *Of what* is this assertion made? *What* is being spoken about? The answer to this question brings out the logical subject of the proposition, which is not always the same as the grammatical subject of the sentence. We then ask, What is the assertion made about this Subject? The answer to this question brings out the logical Predicate and shows whether it is affirmative or negative. The verb must be changed if necessary so as to admit of the Predicate being

introduced by the present tense of the verb "to be." Then we ask whether the predicate is intended to apply to the whole of the Subject, or whether the proposition only intends to commit itself to a statement about "some at least" (which of course covers the special case of "some only"). .In the latter case, the proposition is particular; otherwise, it is universal. We now proceed to give a series of illustrative examples, dwelling rather more fully on some types which frequently occur.

(1) Any proposition of the form "S must be P" (called a "necessary" proposition) is of course logically a universal affirmative: "An equilateral triangle must be equiangular" means that every equilateral triangle can be proved to be equiangular. Similarly, the assertion of impossibility forms an **E** proposition. Some other verbal expressions indicating universality may be mentioned here. In addition to *all*, the following signify a universal affirmative proposition: *every, each, he who, whoever, in every case, always*; just as *no, none, never* signify a universal negative. Even *the* or *a*, when joined to the subject, may signify a universal affirmative. The following are examples: "The old paths are best": logical form, "All well-tried methods are the best methods" "A man's a man for a' that": "All men have the rights and privileges of manhood."

(2) Any proposition of the form "S may be P" (called a "problematic" proposition) merely says "I do not know whether S is P or not"; "the weather may be fine." The nearest in meaning to such a statement, among the four standard forms, is the particular proposition, affirmative or negative. From this point of view, "Some S is P" means that there is *no incompatibility* between S and P; and "Some S is not P" means that there is *no inseparable or necessary connection* between S and P. The following

are examples: " Honesty is compatible with ignorance ": logical form, " Some honest persons are ignorant." " Selfishness may exist without prudence " (*i.e.* there is no necessary connection between selfishness and prudence): *l.f.*, " Some selfish persons are not prudent." " Fine feathers do not make fine birds "; here the contrast is between having " fine feathers " and being " a fine bird," and it is denied that the two facts are necessarily connected: *l.f.*, " In some cases to have fine feathers is not the sign of being a fine bird " (S *o* P). In addition to *some*, the particular affirmative proposition is indicated by *few, many, most, generally, often, sometimes*; and the particular negative by any of these words with a negative.

(3) Statements of the form " All S is not P " (or " Not all S is P," or equivalent expressions) are literary or conversational forms. Their primary logical meaning is " Some S is not P ": " All that glitters is not gold " means logically " Some glittering things are not gold." In this case, " Some " means " some only," and hence the proposition has a secondary implication, " Some glittering things are gold." In exceptional cases, the form " All S is not P," occurring in conversation or writing, signifies an **E** proposition, but this can only be decided by attending to the actual matter of the proposition; thus, " All who act honourably shall not be forgotten " is a universal negative.

(4) Statements of the form " only S is P " frequently occur, in which the Subject is limited by words like *only, alone, none but, none except, none who are not*; " Graduates alone are eligible." The primary meaning of this statement is to exclude non-graduates: *l.f.*, " No non-graduates are eligible." We shall see in the sequel that " All P is S " is an *immediate inference* from " No non-S is P," and

may also be given as the logical equivalent of the given statement: "All eligible persons are graduates."

(5) Any propositions stating a "sign" or "symptom" (the most obvious illustrations are medical) may be regarded as expressing a fact which is found to accompany some other fact—so invariably that we may predicate the one of the other: "S is always accompanied by P," *l.f.*, "All S is P." "Lying is a sure sign of guilt": *l.f.*, "All liars are guilty" (*i.e.* "all who tell lies under accusation are guilty": a proposition which is materially false; but we are now concerned only with its logical form).

(6) Propositions stating a connection between abstract qualities are occasionally found; and in expressing such propositions in logical form they must be referred to cases exemplifying those qualities. "Vice never brings happiness": *l.f.*, "No vicious persons are happy." "Unpunctuality is irritating": *l.f.* "All unpunctual persons are causes of irritation." "Democracy ends in despotism": *l.f.* "All democratic governments are things ending in despotism."

(7) The absence of any mark of quantity generally signifies a universal proposition: this applies in particular to *proverbs* and *current sayings*. In such examples as the following there is little or no doubt:—

Uneasy lies the head that wears a crown.
He can't be wrong whose life is in the right.
The longest road has an end.
Suspicion ever haunts the guilty mind.
Irresolution is always a sign of weakness.
Treason never prospers.

All these are universals. In the following examples the

proposition is no less definitely particular, reducible to the form **I** or **O**:—

Afflictions are often salutary.
Rivers generally run into the sea.
Luck has been known to desert a man.

Sometimes, however, it is really uncertain, from the forms of common speech, whether a universal or a particular proposition is intended. For example:—

Haste makes waste.
Knowledge is power.
Light come, light go.
Left-handed men are awkward antagonists.
Veteran soldiers are the steadiest in fight.

Such sayings in actual speech are often delivered as if they were universals. In the last resort we have to appeal to the facts to decide whether the propositions are really universal: take the logical subject of each proposition—"hasty actions," "men possessing (and using) knowledge," "things lightly acquired"—and then consider whether the attribute is as a matter of fact meant to be predicated of each and every one.

We add a few more examples where there is no doubt as to the "quantity" of the proposition, but the logical structure is not obvious. "It is never too late to mend": *l.f.*, "All men are able sooner or later to improve." "Where no oxen are, the crib is clean": *l.f.*, "Every crib where no oxen are, is clean." "More haste, less speed": *l.f.*, "All cases of excessive haste are cases of little speed," or, say, "of too much haste," "of too little speed." "A little knowledge is a dangerous thing": *l.f.*, "In every case a little knowledge is dangerous" (to say "All little knowledge is dangerous" is logically ambiguous as well as grammatically awkward).

(8) In the case of statements whose Subjects are qualified by indefinite numeral adjectives, such as *few, many, most, any,* we have observed above that in the logical forms these can be expressed only by "some." But in order to secure the least sacrifice of meaning, it is usually advisable to represent such a statement by two particular propositions, one affirmative and one negative. "Few men think about their opinions": *l.f.*, (*a*) "Some men think about their opinions," and (*b*) "Some men do not think about their opinions." When we take into account the ordinary meaning of "few" and the logical indefiniteness of "some," we see that both (*a*) and (*b*) are implied. Usually, however, the logical emphasis is on one of the two propositions rather than the other. Where "few" introduces an affirmative statement, the emphasis is on the negative side; thus, in the example given, the proposition intends to call attention to the number of those who *do not* think about their opinions. Conversely, where "few" introduces a negative statement, the emphasis is on the affirmative side; thus, the proposition "few men are not honest" intends to draw attention to the number of those who *are* honest. "Many were absent": *l.f.* (*a*) "Some were absent," (*b*) "Some were not absent." When great emphasis is laid upon the "many," we may give as *l.f.*, "The number of those who were absent is very large," in which the subject is a Singular collective term. "Many are called but few chosen": *l.f.*, (*a*) "Some who are called are not chosen." (*b*) "Some who are called are chosen." The word "any" is logically ambiguous: it may mean "some" or "all." "Any excuse will not suffice": *l.f.*, "Some excuses will not suffice"; "Any one may judge of B's conduct who examines the evidence": *l.f.*, "All who examine the evidence are able to judge of B's conduct."

The ways in which the real logical structure of assertions or statements is *disguised* in literature or conversation, are innumerable. Even a statement where the grammatical subject is a Singular term is not always a Singular proposition; thus, " Socrates is sometimes not wise " is logically a particular proposition, of which the subject-term is the actions or judgments of Socrates. And even a *question* may be a rhetorical disguise for a definite statement: " What mortal has nothing to regret ? " means logically " All mortals are beings who have some cause for regret (or, something to regret)."

Propositions which appear to be *subjectless* not infrequently occur; sometimes they take an exclamatory form, as " Bad ! " " How hot ! " " Fire ! " Sometimes they take what has been called an " impersonal " form, because the grammatical subject is the " third " personal pronoun, as " It rains," " It thunders." We merely mention these in passing, because their logical analysis belongs to a more advanced stage of the subject. See W. E. Johnson, *Logic*, Vol. I., Ch. II., and Bosanquet, *Logic*, Vol. I., Bk. I., Ch. V.

3. Conditional and Relational Propositions

All the propositions which we have illustrated in the last section have been propositions which made statements without reference to any conditions or limitations under which they are held to be true. We pointed out that propositions so expressed are said to be " categorical " (see page 57). The term is in current use in this sense; thus, a " categorical denial " is understood to mean an absolute or unqualified denial. From *categorical* propositions we distinguished *conditional* propositions, in which P is predicated of S under a condition explicitly stated. Of these there are two kinds:—

(1) *Hypothetical*—

If S is P it is Q
If S is P, Q is R.

(2) *Disjunctive or Alternative*—

S is either P or Q
Either S is P or Q is R.

In a hypothetical proposition, the condition or supposition is introduced by " if " or an equivalent phrase, and is called the "antecedent"; the other part of the proposition, introduced by " then " (expressed, or, more usually, understood) is called the " consequent." We have seen that it has two forms (see page 58). The shortest symbolic expression of the hypothetical proposition is " If A, then C "; this includes both the other forms. Similarly the shortest symbolic expression of the disjunctive or alternative proposition is " Either A or B."

We must, however, examine the normal forms of the hypothetical proposition more closely. The proposition " If S is P it is Q " asserts that a relation between the two qualities P and Q holds universally, so that whenever P is predicated, Q must be also. The implied logical meaning is a connection between the two propositions " S is P " and " S is Q " such that if the former is true the latter follows: " If this report is true, it proves the untruth of what you say." And the proposition " If S is P, Q is R " explicitly asserts a connection between the two propositions, such that if the former is true, the latter follows: " If this report is true, what you say is untrue." These examples suggest what we shall see is the case - that the two forms of the hypothetical proposition are of the same logical nature.

Distinctions of Quantity and Quality in hypothetical propositions may be made by the introduction of the words " always," " never," " sometimes," " sometimes not." Thus " if S is P it is always Q," " if a triangle is equilateral it is always equiangular," corresponds to the form **A** of the categorical proposition; " if S is P it is never Q," " if a triangle is right-angled it is never equiangular," corresponds

to **E**; "if S is P it is sometimes Q," "if a figure is a parallelogram it is sometimes a square," corresponds to **I**; "if S is P it is sometimes not Q," "if a triangle is rectangular it is sometimes not isosceles," corresponds to **O**. But the essence of the hypothetical proposition is the *relation of dependence* which it expresses, and which holds between its antecedent and its consequent. Hence all real hypotheticals are universal. The forms corresponding to **I** and **O** are really categoricals transformed, quite unnecessarily, into hypotheticals. Thus, "if a figure is a parallelogram it is sometimes a square" means "some parallelograms are squares."

We have seen that the hypothetical proposition expresses a relation of dependence between two simpler propositions. When expressed in the hypothetical form, the proposition invites us to attend more to this relation of dependence than to any particular instances of it. But if we attend mainly to the particular instances, actual and possible, to which the proposition may be conceived to apply, then we may express it in the categorical form, the universal affirmative A. Thus, take the proposition, "If S is P, it is Q." When we look at the instances of its application, we see it means that whenever there is a case of S being P, it is also Q. Hence we may express the hypothetical proposition in the form "All S which is P is Q," or "All SP is Q": thus, to take a very simple example, "If iron is impure, it is brittle" may be expressed "All impure iron is brittle."

Coming now to disjunctive propositions, we find that (as in the case of hypotheticals) the two normal forms, "S is either P or Q" and "either S is P or Q is R" are of the same logical nature. But in the case of disjunctives there is an important distinction to be made. Many propositions of the form "either . . . or . . ." merely assert that if one alternative *does not* hold, then the other *does* hold: "All the men in this College either boat or play cricket." "A

good book is valued either for the usefulness of its contents or for the excellence of its style." The whole statement asserts two or more alternatives, *one* of which *must* be true; but there is no intention of denying that both, or more than one, may be true. Such propositions have been called "disjunctive"; but strictly speaking they are not so. The alternatives are not "disjoined" because they are *not mutually exclusive.* Such propositions should be called "alternative propositions."

Disjunctive propositions, properly so called, are those which state alternatives which are mutually exclusive because only one can be true. "You must either pay a fine or go to prison." "A line must be either straight or curved." "This tree is either an oak or an ash." We cannot make an exclusive disjunction about anything unless we have a considerable amount of knowledge about it. Thus, in the case of the three propositions which we have just quoted, the first implies knowledge of the legal bearings of the case; the second implies knowledge of the geometrical meaning of "straight" and "curved" and of the relation between them; the third implies knowledge of two botanical varieties and a comparison of the knowledge with the given instance.

The difference between the alternative and the strictly disjunctive proposition may be further illustrated by expressing each of them as a combination of hypotheticals. We may take the shortest symbolic forms. The proposition "Either A or B," when the alternatives are not mutually exclusive, may be expressed as a combination of *two* hypotheticals:—

$$\begin{cases}\text{If not A, then B,}\\ \text{If not B, then A.}\end{cases}$$

But when the alternatives are mutually exclusive, the

proposition "Either A or B" may be expressed as a combination of *four* hypotheticals:—

$$\begin{cases} \text{If not A, then B,} \\ \text{If not B, then A,} \\ \text{If A, then not B,} \\ \text{If B, then not A.} \end{cases}$$

We shall give further examples of the logical treatment of hypotheticals and disjunctives when we come to the discussion of arguments involving propositions of these forms.

This is the most convenient place in which to call the reader's attention to an aspect of the logical structure of propositions which must be clearly understood; although the questions to which it leads go far beyond the range of an elementary treatment of Logic.

Hitherto we have assumed that logical propositions—by which we always mean, assertions or statements capable of being true or false—naturally express the relation of subject and attribute; so that " S is (or is not) P " means logically that the attributes signified by " P " are affirmed or denied of all or part of the class denoted by " S." We have also seen that most logical propositions can be so interpreted as to express the mutual inclusion and exclusion of classes; so that " S is (or is not) P " means that the class S is included in or excluded from the class P. Now " subject and attribute " and " class inclusion or exclusion " are, of course, kinds of *relation* (between subject and predicate). But logicians had studied these two kinds of relation long before they realised that *other kinds of relation* also find expression in logical propositions, such as relations of cause and effect; of space; of time; of equality, or degree, or comparative magnitude—to mention only these.

For example: every statement in which the predicate is introduced by a *transitive verb* expresses a relation of cause

and effect; and for simple cases of the other propositional relations named, we may give: "Brighton is fifty miles south of London"; "All these problems arose after the War"; "Things which are equal to the same thing are equal to one another"; "Last night was colder than the preceding night"; "Everest is the highest mountain on earth." Such propositions—expressing relations other than those of "subject-attribute" and "class inclusion and exclusion"—have been classed in a special sense as *Relational Propositions.* This special use of the word "relational" need cause no difficulty so long as we remember how it arose: namely, from the fact that the logical nature of propositions of this kind has been fully studied only in recent years. In elementary Logic we are concerned with them only so far as to illustrate some of the simpler forms of inference to which they give rise: this we shall do when discussing Deductive Reasoning. In the meantime we shall find that many of them may be expressed in one or other of the four standard propositional forms without too much distortion of meaning.

Exercise I

It must be distinctly understood that the working of Exercises is as important in Logic as it is in Arithmetic or Geometry. At the stage which we have now reached, practice is needed in the expression of given categorical statements in logical form (*i.e.* in one or other of the four standard forms).

1. Express each of the following propositions in logical form, so as to reproduce as nearly as possible its natural meaning:

(1) All who applaud are not genuine.
(2) What is not practicable is not desirable
(3) Few statements are more deceptive than this
(4) On high mountains the air is pure
(5) All knowledge is only remembrance.
(6) They also serve who only stand and wait.
(7) Only ignorant persons hold such opinions.

(8) For every wrong there is a legal remedy.
(9) Not every advice is a safe one.
(10) Unasked advice is seldom acceptable.
(11) Custom blunts sensibility.
(12) It cannot be that none will fail.
(13) One man is as good as another.
(14) Nothing succeeds like success.
(15) To think, is to be full of sorrow.
(16) We smile not only when we are pleased.
(17) What man has done, man can do.
(18) " What do they know of England who only England know ? "
(19) The eye only sees what the mind can recognise.
(20) Unless help arrives, we are beaten.

2. Distinguish the collective and distributive use of the word *all* in the following:—

(*a*) *Non omnis moriar* (*i.e.* I shall not all die).

(*b*) " All men find their own in all men's good,
And all men join in noble brotherhood."
(*Tennyson.*)

(*c*) *Non omnia possumus omnes* (*i.e.* we cannot all do all things).

(*d*) Sometimes all our efforts fail.

(*e*) *Labor omnia vincit* (*i.e.* labour overcomes all things).

3. Distinguish in the following list the names which are usually (*a*) Singular, (*b*) General, (*c*) Collective. If a name may belong to more than one class, explain and illustrate its uses:—The Bridge of Sighs; city; gold; chain; a pack of cards; an oak tree; the United States; humanity; the centre of the earth; Niagara Falls, a dancing party; the Wall of Hadrian.

4. Are there any names which simply *denote* objects without giving any information about their characteristics ?

5. Is there any kinship of meaning in the use of the word " contrary " of two contrary *propositions* and of two contrary *names* ? Is there any kinship of meaning in the similar use of the word " contradictory " ?

6. Express each of the following statements in logical form, so as to reproduce as nearly as possible its natural meaning:—

(1) Probability is the guide of life.

(2) Men are more to be trusted than we think.

(3) Those transactions are the best to which all the parties freely consent.

(4) A metal bar expands uniformly when it is heated.

(5) Poetic genius and scientific ability are compatible.

(6) Protection from punishment is due to the innocent.

(7) In the last reckoning only character really matters.

(8) The theory of evolution is true

(9) Reason is identical in nature wherever it is found.

(10) To make a good bargain is an advantage.

(11) Wisdom is inseparable from real benevolence.

(12) There is no rule without an exception.

(13) Not all who are called are chosen.

(14) He has no home but Athens.

(15) A few Greeks vanquished the vast army of Darius.

(16) Two wrongs do not make a right.

(17) All these claims upon my time overpower me.

(18) Men are not what they were.

(19) " An honest man's the noblest work of God."

(20) What makes and maintains a State is the idea of a common good.

4. A Modern Classification

The study of the great *variety of relations* which can be expressed in logical propositions has led recent logicians to work out a classification of logical propositions which differs materially from the traditional classification,[1] and which in contrast therewith may be called " modern."

In the course of its history the traditional four-fold classification has come to mean that every categorical proposition is of the subject-attribute type. The predicate

[1] Students who do not intend to carry their study of Logic beyond the elementary stage may omit this Section.

is regarded as *adjectival*, and is expressed by an adjective or a group of words equivalent to an adjective. Thus, in such a proposition as "all men are fallible," the predicate is quite naturally regarded as an attribute of every member of the class "men"; but in such a proposition as "Brutus killed Caesar," the predicate "having killed Caesar" has also to be regarded adjectivally as an attribute of Brutus. For certain purposes it is found convenient to treat the predicate also as a class, marked out by the stated attribute. "All the class *men* is included in the class *fallible beings*"; but the primary meaning of the predicate is taken to be adjectival. In recent Logic this limitation is definitely rejected.

Aristotle's own view of predication was much wider than this. The idiom of the Greek language made it natural for him to express *any* logical proposition, say, "all men are mortal," in the form "mortality *is predicated of* all men": and when he said "is predicated of" he was aware that the words cover many different kinds of relation, and not only the noun-adjective relation. Whereas, in Latin and in modern European languages, the wide and vague use of the verb "to be" naturally suggests a merely adjectival interpretation of the predicate The Aristotelian view may be expressed in modern English by saying that in the logical proposition a Subject or a group of Subjects *is characterised* in certain ways in the predicate, which may or may not be adjectival in the grammatical sense. Relational propositions of the two-term form can be interpreted in this way without any difficulty.

There is, however, another feature of the Aristotelian doctrine, which has had great influence on the traditional treatment of the logical proposition Aristotle believed that "classes"—groups of individuals related by having in common certain essential qualities—are of fundamental importance because they belong to the very structure of the world. He was therefore led to believe that a proposition, except when it refers exclusively to a particular individual, always refers to a class, and predicates (affirms or denies) something of all members or some members of that class.

We shall first summarise the modern classification, and

then explain it as simply as possible. There are three principal types:—

(1) Simple propositions, *i.e.* those whose constituents are not themselves propositions. These include:—

- (*a*) Subject-attribute propositions.
- (*b*) Relational propositions (having two, three, or more Terms).
- (*c*) Class-membership propositions, assigning an individual to a class, and forming a transition to (2).

(2) General propositions, stating relations between classes, and in the two-term form introduced by "all" or "some."

(3) Compound propositions, in which the constituents are themselves propositions. These include:—

- (*a*) Conjunctive propositions;
- (*b*) Implicatives;
- (*c*) Alternative propositions;
- (*d*) Disjunctives.

(1) (*a*) Take a **Simple Proposition,** such as "this rose is yellow" (or, "is not red"). The former statement asserts a connection between two things or objects of thought, "this rose" and "yellow"; the latter denies or excludes another connection which was supposed or questioned. For convenience, keep to the affirmative form. This particular flower in the real world is asserted to possess that quality; and the proposition is evidently of the subject-attribute type. In the example given, the familiar noun-adjective form is sufficient; but this is not always the case. "The last Emperor of Rome was utterly incompetent and helpless": here the logical subject is grammatically a phrase, and the predicate is also a phrase. But the proposition is logically of the subject-attribute form.

Again: "The efforts of the veterans were unavailing against such odds": here the predicate is a somewhat elaborate phrase of an adjectival or attributive character. Many such examples of two-term propositions in the subject attribute form, will occur to the reader.

(*b*) Simple propositions are by no means all of the subject-attribute type. There are numberless propositions of the two-term form in which the relative factor is a transitive verb; or in which it expresses a spatial or temporal relation, or one of equality, or degree, or comparative magnitude—to mention only these. The reader will easily identify many kinds of relation expressible in two-term propositions. For a few examples, see the present Chapter, page 75; and also Chapter I., page 12. These are called **Relational Propositions**, and are to be distinguished from the subject-attribute propositions and classifying propositions which we have already discussed.

Theoretically there is no limit to the number of terms which a relational proposition may contain. For example: "Gainsborough lies between Lincoln and Doncaster": here the three terms are obvious, and the relating factors are "and," "between." Again: "The Chairman asked the Treasurer to oppose the Resolution demanding the Secretary's resignation". here we have four terms, and the relating factors are, "asked" "to oppose," "demanding." Again: "The priests gave Judas Iscariot thirty pieces of silver": here the relating factors are, "gave," introducing the grammatically "direct object," and "to" (understood), introducing the grammatically "indirect object"—"Judas." We shall not give further examples. The logical treatment of propositions containing three or more terms is beyond the range of an elementary work. The most difficult problems arise out of the analysis and classification of the relating factors.

(*c*) A specially important kind of two-term proposition remains to be indicated. It is the kind of proposition which affirms (or denies) that the individual person or "thing" specified in the subject belongs to or is a member of a class specified in the predicate. "Socrates was a

philosopher " means logically that Socrates is a member of the class " philosophers." " That dog is a retriever " means that he belongs to the class of dogs called " retrievers." Such propositions are of the two-term form, and the subject is a Singular term. They may be called **Class-membership Propositions.**

(2) Propositions which assign an individual to a class must be carefully distinguished from what have been called **General Propositions,** which state relations *between classes.* The simplest form of the General Proposition is the two-term form, in which it is asserted that *some* or *all* of a class specified in the subject is included in or excluded from another class specified in the predicate. " All Athenians were Greeks," *i.e.* the whole of the class " Athenians " is included in the class " Greeks." " Some Germans are poets," *i.e.* the class " Germans " is partly included in the class " poets."

(3) Hitherto we have been speaking of Simple propositions, as defined above. We shall now give examples of **Compound Propositions,** *i.e.* those whose constituents are themselves propositions. The most important types are these:—

(*a*) **Conjunctive Propositions** in which the whole statement consists of two or more propositions connected by the conjunction "and," where the "and" does not merely set the propositions side by side but implies some kind of connection between them, as in the familiar story " Jack fell down *and* broke his crown." When the " and " simply sets the propositions side by side, as " I was writing all the morning and my neighbour was working in his garden," we have two separate propositions, not a single conjunctive proposition. The really conjunctive

proposition gives us the joint logical force of the constituent propositions.

(*b*) **Implicative Propositions.** These are the "hypothetical" propositions which we discussed above. We there pointed out that any such proposition asserts a relative of dependence between two simple propositions. Hence modern logicians call them "implicative" propositions. The "antecedent" *implies* the "consequent"; the antecedent, the *implying* proposition, is introduced by "if" or an equivalent phrase; and the consequent, the *implied* proposition, is introduced by "then" (expressed or understood). For examples, see page 72.

(*c*) **Alternative Propositions.** These are the so-called "disjunctive" propositions in which the alternatives are *not mutually exclusive.* For examples, see page 74.

(*d*) **Disjunctive Propositions,** properly so called, in which the alternatives are mutually exclusive. For examples, see page 74.

The importance of the preceding classification is to some extent concealed by the unavoidable simplicity of the examples which we have given. It is based, with slight modifications of terminology, on the work of Bertrand Russell and W. E. Johnson; see, for example, Johnson, *Logic*, Vol I, Ch III. (on "Compound" propositions); and the references to Russell in Stebbing, *Modern Introduction to Logic*, Ch VIII. and IX.

From the point of view of the philosophical theory of knowledge, an elaborate classificatory system of the forms of logical Judgment has been worked out by Bosanquet, *Logic*, Vol. I., Ch. V., VI., VIII. Bosanquet's work is historically important, in reference both to its antecedents in German thought and its influence on the study of Logic in England and America during the last fifty years. From a point of view not identical with that of Bosanquet, but also as a contribution to the theory of knowledge, the German logician Sigwart had previously expounded a classification of the forms of Judgment in his *Logic* (see the Eng. trans., Vol. I., Ch. III., IV., V.).

CHAPTER V

IMMEDIATE INFERENCE

We now come to a topic which forms an introduction to the Logic of Inference, and at the same time is closely related to the subject of the previous chapter.

1. Immediate Inference: (i) By Opposition

When we have before us a single proposition expressed in one or other of the four standard forms, it is possible to infer, directly from the given proposition, other propositions *implied* by it. These implied propositions are *equivalent propositional forms.* The equivalent propositional forms are not mere verbal changes—*i.e.* they are not mere restatements (in different words) of the same relation between S and P, and they are not "new" propositions in the sense of being simply different from the given proposition. They express other "sides" or "aspects" of the relation stated in the given proposition. And as they can be derived directly from the given proposition, they are classed together under the name of "Immediate Inference."

Immediate Inference is of two kinds: (1) Inference by or through what is technically termed **Opposition,** and (2) Inference by or through what is technically termed **Eduction.** In explaining these elementary processes, it is convenient to regard the propositions with which we are dealing as expressing relations between classes. We

have seen that some propositions actually bear this interpretation as their primary logical meaning; but it is a possible way of regarding most propositions, and it is the simplest way for the subject now before us.

We therefore understand the four standard propositional forms in the way explained above (see Chapter IV., page 62), and which for convenience may be summarised here:—

Proposition **A**: The individual or the whole class denoted by the subject falls within the class denoted by the predicate.

Proposition **E**: The individual or the whole class denoted by the subject falls outside the class denoted by the predicate.

Proposition **I**: *Some at least* of the class denoted by the subject falls within the class denoted by the predicate.

Proposition **O**: *Some at least* of the class denoted by the subject falls outside the class denoted by the predicate.

Now when we speak technically of the "opposition" of two propositions, both of which are expressed in strict logical form, and both of which have the same subject and predicate, we have in view the following relations (only six being possible):—

(1) Universal affirmative and particular negative: S *a* P and S *o* P.

(2) Universal negative and particular affirmative: S *e* P and S *i* P.

(3) Universal affirmative and universal negative: S *a* P and S *e* *P*.

(4) Particular affirmative and particular negative: S *i* P and S *o* P.

(5) Universal affirmative and particular affirmative: S *a* P and S *i* P.

(6) Universal negative and particular negative: S *e* P and S *o* P.

In (1), "All S is P" and "Some S is not P," the propositions are called *contradictories*. They cannot both be true, and they cannot both be false; one must be true and the other false. For example, if "All S is P" is false, this means that not all the class S falls within the class P, *i.e.* some (at least) of it must fall outside P; that is, "Some S is not P" is true; and so on.

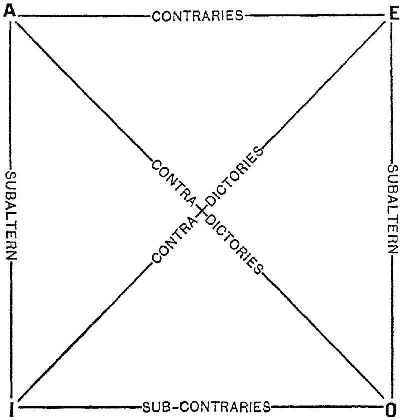
In (2), "No S is P" and "Some S is P" the propositions are likewise *contradictories*; one must be true and the other false.

In (3), "All S is P" and "No S is P," the propositions are called *contraries*. They cannot both be true; but they may both be false; for example, the class S may fall partly within the class P (so that "No S is P" is false) and partly outside it (so that "All S is P" is false).

In (4), "Some S is P" and "Some S is not P," the propositions are called *subcontraries*. The "opposition" is merely technical, since both may be true. But both cannot be false; for if so, the class S must be all within the class P, since S *o* P is false, and at the same time all outside it, since S *i* P is false; and this is impossible.

In (5) and (6), "Some S is P" is called the *subaltern* of "All S is P," and "Some S is not P" the *subaltern* of "No S is P." Here again the "opposition" is merely technical. In each case, both may be true, for the truth of the universal implies the truth of the particular. But if we only know the truth of the particular propositions, *i.e.* if we only know that "some (at least) of S is P," or that "some (at least) of S is not P," we do not know whether the respective universals are true or not.

These six relations are represented in the following diagram, traditionally called the "Square of Opposition":—



From these relations, the following Immediate Inferences may be derived (the reader will see that in order to get Immediate Inference by "Opposition," we must start by taking the given proposition as true, or false, as the case may be):—

(1) If **A** is true, then **I** is true, and **E** and **O** are false.
(2) If **A** is false, then **I** and **E** are doubtful, and **O** is true.
(3) If **E** is true, then **A** and **I** are false, and **O** is true.
(4) If **E** is false, then **I** is true, and **A** and **O** are doubtful.
(5) If **I** is true, then **E** is false, and **A** and **O** are doubtful.
(6) If **I** is false, **A** is false, and **E** and **O** are true.
(7) If **O** is true, **A** is false, and **E** and **I** are doubtful.
(8) If **O** is false, then **A** and **I** are true and **E** is false.

These results may be thus summarised in tabular form (the letters T, F, D stand respectively for "true," "false," "doubtful"):—

	A *is*	**E** *is*	**I** *is*	**O** *is*
Given **A** true ..	(T)	F	T	F
,, **A** false ..	(F)	D	D	T
,, **E** true ..	F	(T)	F	T
,, **E** false ..	D	(F)	T	D
Given **I** true ..	D	F	(T)	D
,, **I** false ..	F	T	(F)	T
,, **O** true ..	F	D	D	(T)
,, **O** false ..	T	F	T	(F)

The reader will observe that the lower half of the square *reflects* (not *repeats*) the upper half; thus, compare the first and eighth lines, the second and seventh, the third and sixth, the fourth and fifth.

2. Immediate Inference: (ii) Eduction

In the case of "Eduction," we derive propositions which are strict formal equivalents of the given proposition; that is, equivalents whether the given proposition is true or not.

There are two fundamental processes of **Eduction.** These are, (*a*) Conversion, by which we obtain an equivalent proposition in which S and P have changed places; and (*b*) Obversion, in which the equivalent proposition has for predicate the contradictory term "non-P" instead of P.[1] All other processes of Eduction

[1] In some recent textbooks the symbol P′ is used to indicate the logical contradictory of any term P.

consist of an alternate performance of these two elementary operations.

(1) **Conversion** is the process by which from a proposition of the type SP we infer an equivalent proposition of the type PS. The given proposition states a class-relation from the side of S, and the *converse* states the same relation from the side of P. Hence (*a*) the quality (affirmative or negative) of the original proposition is unchanged in the converse; and (*b*) no term must be distributed in the converse which was not distributed in the proposition given to be converted (sometimes called the " convertend "). The first rule is a matter of definition. The second rule follows from the meaning of the " distribution " and " non-distribution " of terms in the four standard propositional forms; the logical converse must not *say more* than is said by the form of the " convertend." Our conclusions are therefore as follows.

In the converse of " All S is P " S and P change places, and P has the mark of quantity instead of S. Now in the **A** proposition, " All S is P," we only know that some (at least) of P is referred to: hence we cannot say more than this in the converse: " Some P is S." Thus, the logical converse of " All men are fallible " is " Some fallible beings are men." There may be " fallible beings " other than " men "; the original proposition tells us nothing as to this. For a similar reason, the logical converse of " Some S is P " is " Some P is S "; the converse of " Some men are learned " is " Some learned beings are men." But in the proposition " No S is P " it is explicitly stated that the two classes fall outside one another; both terms are distributed, and the logical converse is " No P is S." Converting " No men are perfect " we get " No perfect beings are men." Finally, in the case of the particular

negative proposition " Some S is not P " we cannot get a formal logical converse. The proposition says that " Some (at least) of the class S falls outside the class P." Try the converse: " What falls outside the class P is some (at least) of the class S." But the subject of this proposition has no mark of quantity. We must be able to say how much of what is outside is referred to (*i.e.* either " all " or " some at least "); but the original proposition does not enable us to do this. We observe that two propositions of the forms " Some S is not P " and " Some P is not S " may both be true materially or as a matter of fact; thus, " Some metals are not brittle " and " Some brittle things are not metals " are both true, but the latter is not known by mere logical conversion of the former—it is derived from our knowledge of metals and of brittle things. And in many cases such propositions are not both true; the attempt to convert a particular negative may lead to results which are materially false or even absurd. Thus, from " Some Europeans are not Frenchmen " we cannot infer that " Some Frenchmen are not Europeans "; and from " Some candidates who sit for an examination do not pass it " we cannot infer that " Some candidates who pass an examination do not sit for it."

Looking back on the foregoing discussion, we see that in the case of the propositions **E** and **I** we *lose none* of the logical meaning of the forms when we convert them; in other words, we can recover the original proposition by re-converting the converse: S *e* P—P *e* S—S *e* P; S *i* P—P *i* S—S *i* P. In both these cases the process is called *simple conversion*. But in the case of the **A** proposition, we *lose part* of the meaning in conversion, owing to the indefiniteness of the logical " some "; and we cannot recover the original proposition by re-converting: S *a* P—P *i* S—S *i* P. In the case of the **A** proposition, the process

is called *conversion by limitation* (*per accidens*). The following is a summary statement of our results:—

A converts *per accidens*;
E converts simply;
I converts simply;
O does not convert.

(2) **Obversion** is the process by which we pass from a proposition of the form SP to an equivalent proposition of the form "S non-P." We pass from an affirmative to a negative statement of the same truth. The rule is: change the "quality" of the proposition and substitute for the predicate its logical contradictory non-P.

Original propositions	*Obverses*
All S is P (**A**)	No S is non-P (**E**)
No S is P (**E**)	All S is non-P (**A**)
Some S is P (**I**)	Some S is not non-P (**O**)
Some S is not P (**O**)	Some S is non-P (**I**)

In Obversion, it is desirable, in order to secure neatness in the logical form, to substitute a single term for the contradictory predicate non-P, when such a term exists. It will of course be one of the *negative terms* of ordinary language. But care must be taken not to use a *contrary* for a *contradictory*; *e.g.* "unhappy" for the contradictory of "happy." In the following cases, ordinary language provides a suitable contradictory:—

Original propositions	*Obverses*
All men are fallible (**A**)	No men are infallible (**E**)
No men are perfect (**E**)	All men are imperfect (**A**)
Some men are learned (**I**)	Some men are-not unlearned (**O**)
Some men are-not trustworthy (**O**)	Some men are untrustworthy (**I**)

Sometimes the phrase "other than" is useful in forming the contradictory predicate.

It is easy to see that Obversion produces a formally equivalent proposition. If all the class S falls within the class P, then none of S can fall outside P; *i.e.* "All S is P" and "No S is non-P" are equivalent; if none of S falls within P, then all of S falls outside P; *i.e.* "No S is P" and "All S is non-P" are equivalent; if some of S falls within P, then that part of S cannot fall outside P; *i.e.* "Some S is P" and "Some S is-not non-P" are equivalent; and if some of S does not fall within P, then that part of S falls outside P; *i.e.* "Some S is-not P" and "Some S is non-P" are equivalent.

We shall now give some examples of Conversion and Obversion.

(*a*) "Only Protestant princes can sit upon the throne of England." Now we have seen (page 67) that a proposition of the form "only S is P" is equivalent to "No non-S is P" *or* "All P is S." The latter form is often more convenient, as here: *l.f.* "All persons who can sit upon the throne of England are Protestant princes"; *converse:* "Some Protestant princes are persons who can sit upon the throne of England"; *obverse:* "No persons who can sit upon the throne of England are *other than* Protestant princes."

(*b*) "No one is free who cannot command himself": *l.f.* "None who cannot command themselves are free"; *converse:* "No free men are those who cannot command themselves"; *obverse:* "All who cannot command themselves are unfree" (the most convenient contradictory of "free" is the negative term "unfree").

(*c*) "Mercy but murders, pardoning (*i.e.* if it pardons) those that kill": *l.f.* "All mercy which pardons those

that kill is murderous"; *converse:* "Something murderous is mercy which pardons those that kill"; *obverse:* "No mercy which pardons those that kill is *other than* murderous."

(*d*) "In man there is nothing great but mind." This is an "exceptive" proposition: *l.f.* "Nothing other than mind is great in man"; *converse:* "Nothing great in man is other than mind"; *obverted converse:* "All that is great in man is mind" (here the double negative, "not-other than mind," equals a positive).

(*e*) "No admittance except on business." Another "exceptive" proposition: *l.f.* "None other than those on business are admitted"; *converse:* "None admitted are other than persons on business" (or, "are persons not on business"); *obverted converse:* "All admitted are persons on business."

(*f*) "All really virtuous men are happy": *l.f.* as given; *converse:* "Some who are happy are really virtuous"; *obverse:* "No really virtuous men are other than happy."

(*g*) "Some love England and her honour yet." Here the emphasis is on the fact that such persons exist: *l.f.* "Some lovers of England's honour are still to be found"; *converse:* "Some persons still to be found are lovers of England's honour"; *obverted converse:* "Some persons still to be found are not other than lovers of England's honour."

We have seen that Singular propositions are traditionally classed as universals, and have to be treated accordingly. "Brutus murdered Caesar": *l.f.* "Brutus is one who murdered Caesar" (see note on the relation of the copula to *time*, page 63); *converse:* "Some one who murdered

Caesar is Brutus"; *obverse:* "Brutus is-not other than a murderer of Caesar." "Oxford is an ancient university": *l.f.* as given; *converse:* "*Some one* of the ancient universities is Oxford"; *obverse:* "Oxford is-not other than an ancient university" (**A**). "Britain is an island": *l.f.* as given; *converse:* "Some one of the islands is Britain"; *obverse:* "Britain is-not other than an island." When both subject and predicate are Singular terms, we can convert the proposition simply: "St. Andrews is the oldest university in Scotland": *l.f.* as given; *converse:* "The oldest university in Scotland is St. Andrews"; *obverse:* "St. Andrews is-not other than the oldest university in Scotland."

(3) Other processes of Eduction consist in the application of conversion and obversion alternately. Two such processes must be explained: "Contraposition" and "Inversion."

Contraposition is the process by which we pass from a proposition of the type SP to one of the type "non-P S"; the derived proposition, called the "contrapositive," has the contradictory of the original predicate for its subject, and the original subject for its predicate. The rule is: obvert the given proposition, and then convert the proposition thus obtained. In each of the following examples, (1) is the given proposition, (2) its obverse, and (3) its contrapositive:—

(1) All S is P (**A**)	(1) No S is P (**E**)
(2) No S is non-P (**E**)	(2) All S is non-P (**A**)
(3) No non-P is S (**E**)	(3) Some non-P is S (**I**)
(1) Some S is P	(1) Some S is-not P
(2) Some S is-not non-P (**O**)	(2) Some S is non-P
(3) *None*	(3) Some non-P is S

For example: "All men are fallible" yields as its contrapositive "No non-fallible beings are men"; "Some men are learned" yields no result, because its obverse is an **O** proposition and cannot be converted; "No men are perfect" yields "Some non-perfect beings are men": and "Some men are not trustworthy" yields "Some non-trustworthy beings are men."

(4) **Inversion** is the process by which we pass from a proposition of the type SP to a proposition which has the same predicate, but for its subject the contradictory of the original subject. Starting with the given proposition, we apply conversion and obversion alternately until we either reach the required result (a proposition with non-S as its subject-term), or are brought to a standstill by a proposition which cannot be converted. It will be found that an Inverse is obtainable only when the original proposition is universal. Thus:—

Given: All S is P (**A**)
Obvert: No S is non-P;
Convert: No non-P is S;
Obvert: All non-P is non-S;
Convert: Some non-S is non-P;
Obvert: Some non-S is-not P.

The last proposition is the required Inverse. In the case of an **A** proposition, we must begin with obversion. In the case of an **E** proposition, we begin with conversion:—

Given: No S is P (**E**);
Convert: No P is S;
Obvert: All P is non-S;
Convert: Some non-S is P.

The last proposition is the required Inverse. In the case of an **I** or an **O** proposition, we come to an inconvertible

result before any Inverse is obtained. We may sum up the rules for Inversion thus:—

(1) To "invert" an **A** proposition, obvert and convert alternately;

(2) To "invert" an **E** proposition, convert and obvert alternately.

The following examples are illustrative of all the processes of Eduction:—

(*a*) *Given:* Every truthful man is trusted (**A**);
Converse: Some trusted men are truthful (**I**);
Obverted converse: Some trusted men are-not untruthful (**O**).

(*b*) *Given:* No cultivated district is uninhabited (**E**);
Converse: No uninhabited district is cultivated (**E**);
Obverted converse: All uninhabited districts are uncultivated (**A**).

(*c*) *Given:* Some British subjects are dishonest (**I**);
Converse: Some dishonest persons are British subjects (**I**);
Obverted converse: Some dishonest persons are-not aliens (**O**).

(*d*) *Given:* Every poison is capable of destroying life (**A**);
Obverse: No poison is incapable of destroying life (**E**);
Contrapositive: Nothing incapable of destroying life is poison (or, is poisonous) (**E**);
Obverted contrapositive: Everything incapable of destroying life is non-poisonous (**A**).

(*e*) *Given:* Some unjust laws are-not repealed (**O**);
Obverse: Some unjust laws are unrepealed (**I**);
Contrapositive: Some unrepealed laws are unjust (**I**);
Obverted contrapositive: Some unrepealed laws are-not just (**O**).

(*f*) *Given:* No unjust act is worthy of praise (**E**);
Inverse: Some just acts are worthy of praise (**I**);
Obverted inverse: Some just acts are-not unworthy of praise.[1]

All the forms of Eduction may be thus set forth in tabular form (P′ = non-P):—

			A	E	I	O
i.	1	Original Proposition ..	S *a* P	S *e* P	S *i* P	S *o* P
	2	Obverse of (1)	S *e* P′	S *a* P′	S *o* P′	S *i* P′
ii.	3	Converse of (1)	P *i* S	P *e* S	P *i* S	
	4	Obverted Converse of (1) ..	P *o* S′	P *a* S′	P *o* S′	
iii.	5	Contrapositive of (1) ..	P′ *e* S	P′ *i* S		P′ *i* S
	6	Obverted Contrapositive of (1)	P′ *a* S′	P′ *o* S′		P′ *o* S′
iv	7	Inverse of (1)	S′ *o* P	S′ *i* P		
	8	Obverted Inverse of (1) ..	S′ *i* P′	S′ *o* P′		

Immediate Inference is not a trivial matter. It is of practical importance. In the interpretation of legal documents, rules, etc., the real implications of the statements will be much more evident if we remember these logical processes. In ordinary thought we are constantly making mistakes through neglect of them: taking "Only S is P," for instance, as if it implied that "All S is P," or converting an **A** proposition simply. From "ill-doers are ill-dreaders" (understood as universal) it is easy to slip into the simple or unlimited converse "ill-dreaders are ill-doers (also understood as universal). Similarly "all

[1] For this set of examples I am indebted to Welton, *Manual of Logic*, Vol. I., pp. 262-6.

beautiful things are agreeable" may be true, but it does not follow that "all agreeable things are beautiful." The two propositions "All S is P" and "All P is S" may be true together; but if so, they have to be proved independently from the subject-matter to which they refer. Neither of them is logically inferrable from the other. We must add that the same is true of the two propositions "All S is P" and "No non-S is P"; they *may* both be true; but, in this case also, neither is logically inferrable from the other.

Exercise II

1. Give all the Immediate Inferences which can be derived from the proposition "No idle person is deserving of success." (In dealing with a problem of this type, it is desirable first to give the inferences by Opposition, when the given proposition is taken (*a*) as true, and (*b*) as false; and then to give the Eductions.)

2. Show how to obtain the Converse of the Contrary of the Contradictory of the proposition "Some crystals are cubes." How is it related to the original proposition?

3. Is it the same thing to affirm the falsity of the proposition "Some S is P," and the truth of the proposition "Some S is not P"? Give your reasons.

4. "Some political organisations ought to be condemned." Can you upon any principle draw the inference "Some political organisations ought to be commended"?

5. What is the logical relation between each of the following propositions and the proposition "All crystals are solids"?

   (*a*) Some solids are-not crystals.

   (*b*) Some non-crystals are-not solids

   (*c*) No crystals are non-solids.

   (*d*) Some solids are crystals.

   (*e*) Some non-solids are-not crystals.

   (*f*) All solids are crystals.

(Here, "logical relation" may be taken to include the relations given in the square of Opposition and the table of Eductions.)

6. (*a*) Express the substance of the following conversation in logical form, and state the relations between the propositions:—

X.: " None but fools would spend half their income in rent."

Y.: " And not every fool."

X.: " I know some who do."

(*b*) What is the logical relation between the propositions " He that is not against us is for us " and " He that is not for us is against us " ?

7. Give, where possible, (*a*) the converse, (*b*) the obverse, (*c*) the contrapositive, of each of the propositions in Exercise I., Questions 1 and 6 (pages 76-7). First obtain the strict logical form of each proposition.[1]

3. The Laws of Thought

There are important ambiguities in the meaning of the term " Law," which must be carefully noted before we can examine the appropriateness of the expression " Laws of Thought "—which is one of those handed down to us through the traditional treatment of Elementary Logic.

In one sense of the word, we speak of " Laws of Nature," which are general statements of what uniformly happens (under conditions supposed to be known). A single exception to such a law would make it no longer a Law of Nature. A Law of Nature may appear to be " violated," when in reality it is suspended or counteracted by some other Law, as when the solidity of a table prevents a book from falling to the ground under the gravitational attraction of the earth.

In another sense, a Law is a precept or rule laid down by some authority—an injunction or command addressed to persons who are called upon to obey it but have it in their power to disobey. This use of the term is seen in such expressions as " the Law of the land," " the Law of conscience."

[1] In all such exercises, the student should carefully avoid cumbrous or uncouth forms of expression. It is possible to be logical without murdering " the King's English."

Neither of these meanings of the term "Law" is appropriate in the case of Thought or Reasoning, and its use in this connection has given rise to unnecessary controversy. For the present it will be sufficient if the reader regards the expression "Laws of Thought" as a historic label for certain *first principles* which are the foundation of all reasoning, in the sense that consecutive thought and coherent argument are impossible unless they are taken for granted. "Axioms of Reasoning" would be a more appropriate name than "Laws of Thought."

An **axiom** may be defined as "a general proposition which is incapable of proof and stands in no need of proof"—that is, of proof by inference from other propositions or from sense-observation. All proofs must ultimately, or "in the last resort" or "at bottom," start from *some* propositions which are received as true without proof; otherwise we could never attain to any kind of proof until we became absolutely omniscient. Some propositions are of so self-evident a character that they must be accepted as true by every rational mind immediately the terms in which they are expressed are understood. Such propositions are axioms.

Three "Axioms of Reasoning" have been recognised as of such primary and fundamental importance that they have been called, above all other axioms of Logic, "the Laws of Thought." The three principles are these:—

(1) The Law of Identity: "Everything is what it is," or "A is A."

(2) The Law of Contradiction: "A thing cannot both be and not be so-and-so"; or, "A is not both B and not B."

(3) The Law of Excluded Middle: "A thing either is or is not so-and-so"; or, "A either is or is not B."

(1) The **Law of Identity** has its primary application to the meanings of logical terms. The *same term must have the same meaning*—the same for different minds, and for one mind at different times. A term must have a fixed meaning, clear in itself and distinct from other meanings. If the meaning of a term is changed, it should be done deliberately and for a sufficient reason. This is a *logical ideal* rather than a psychological fact or characteristic of our actual thinking. We called attention to this when speaking of Definition (page 41).

The Law of Identity may also be taken as expressing the nature of the logical proposition. So regarded, its interpretation is a problem famous in the history of Philosophy. But there is an interpretation of it, in reference to propositions with two constituent terms, which is intelligible and sufficient for our present purpose. The typical two-term proposition seems to say, not that something "*is* itself," but that it "*is* something else", not that "S is S" or "P is P," but that "S is P" On the other hand, while the proposition distinguishes S and P, it asserts (assuming it to be affirmative) that they are in some way *united*. Now this combination of difference with unity is possible because any typical logical *term* has the two-fold meaning, *connotation* and *denotation*, in the sense in which we have previously defined these words. Looked at from the side of connotation, the proposition distinguishes S and P, looked at from the side of denotation it unites them: and the two sides are inseparable. This is evident alike in any proposition whose subject-term is the name of a class and in any proposition whose subject-term is a Singular name.

(2) The **Law of Contradiction** may be interpreted so as to refer (*a*) to the meaning of a term, (*b*) to the mutual consistency of propositions.

(*a*) Just as the Law of Identity secures the identical reference of a term to a meaning, so the Law of Contradiction secures the same result by forbidding a term to be diverted to another meaning in the same argument or discourse.

(*b*) In reference to the mutual relations of *propositions*, the Law of Contradiction declares that *incompatible* propositions cannot be true *together*. We have already seen that the simplest case of incompatibility is between a proposition and its logical contradictory. Now when the subject-term of a proposition is Singular, contradictory opposition takes the form " S is P—S is not P ": " this line is longer than that—this line is not longer than that "; and it is this kind of incompatibility which the Law of Contradiction has specially in view. In fact, it was first formulated to meet the case of attempting to affirm and deny the same predicate of the same subject. But it applies equally to the contradictory class-propositions (" S *a* P—S *o* P," and " S *e* P—S *i* P ") and also to the contrary class-propositions (" S *a* P—S *e* P ") whose relations are shown in the Square of Opposition.

Sometimes, in stating the Law of Contradiction, a reference to time and other conditions is introduced: " a thing cannot both be and not be so-and-so *at the same time and in the same respect* (*e.g.* at the same point). The explicit reference to such conditions is useful; but it is not logically necessary, because they are implied in the *meaning* of the proposition whose truth and falsity is in question. The qualities of the same object may be different at different times and places, but such changes do not conflict with the Law of Contradiction as a logical axiom.

(3) The **Law of Excluded Middle** is a further " specification " of the Law of Contradiction.

(*a*) In reference to the meanings of *terms*, it declares any given meaning either does or does not belong to a given term in a given context. It thus supplements the rule that the meaning of a term must be definite and must remain the same in the same argument or discourse.

(*b*) In reference to the mutual consistency of *propositions*, it declares that of two contradictories one must be true and the other false. Taken together with the Law of Contradiction, it provides a definition of contradictory opposition; and (like the Law of Contradiction) its primary reference is to the opposition of Singular propositions ("S is P—S is not P"); but it is illustrated also in the contradictory class-propositions shown in the Square of Opposition. On the other hand, it does not apply at all to propositions affirming *contrary predicates* of an object; for example, "this opinion is entirely true—this opinion is entirely false"; here there is a third possibility—the opinion may be a mixture of truth and error. And in like manner it does not apply at all to the opposition of the logically contrary propositions ("S *a* P—S *e* P"), which may both be false.

The reader will see on consideration that the whole doctrine of Immediate Inference, as expounded above, is based upon these "Laws of Thought." The main principle of logical conversion, that no term may be distributed in the converse which was not distributed in the "convertend," is an application of the Law of Identity. We retain the indefinite extent of the undistributed predicate, in the conversion of **A** and **I**, by using the indefinite "some" in the subject of the converse; we retain the definite extent of the distributed predicate, in the conversion of **E** by using the definite "no" in the subject of the converse. And in both obversion and conversion, the use of the expressions "included in" and "excluded from," in the class-interpretation of the propositions (see for example our illustration of the formal validity of obversion, page 92), exemplifies the Laws of Contradiction and Excluded Middle.

CHAPTER VI

DEDUCTIVE REASONING

In our introductory chapter we gave illustrative "samples" of actual processes of reasoning or inference[1] in some of their simplest forms (see Chapter I., pages 13-14); and we distinguished three different types, which we provisionally described as reasoning, (1) from "particular to general"; (2) from "particular to particular"; (3) from "general to particular."

1. Meaning of Deduction

The process of reasoning consists in the combination of given facts or propositions, which for the present we will call data ("things given"), and which when combined yield a conclusion which is *new* in the sense that it is not inferrable from any of the data taken separately. Now it is evident that the combination of the data cannot be made at random, as marbles might be drawn from a bag. We need a guiding thread; and from the characteristic of deductive reasoning is, that *we start with the guiding thread in our hands.*

We may put this in another way, introducing the important logical term "system." Reasoning consists in arranging our data in some kind of system; and in **deductive reasoning** we start knowing what the nature of the system is. What is a "system"? We must grasp the general idea of it, when used as an abstract term. A system is a whole in which every part has a definite relation

[1] We shall use "reasoning" and "inference" as synonymous terms, employing either word as may be convenient.

of *dependence* upon every other, and all these relations of dependence show an orderly plan. In other words, a "system" is an arranged or planned order of mutual *dependence*. When we describe the "order" as "arranged" or "planned," we are not saying anything of its source or origin, but only of its character. The parts depend upon one another and upon the whole, and the whole depends upon the parts. There is an essential thread of relation binding each to each and all together. Imagine a heap of bricks. It forms a whole in which the bricks are the separate parts; but the position of each in reference to the others is a matter of indifference. A heap is not a system. But imagine the bricks arranged as a wall or an arch: here there is a planned order of mutual dependence among the parts. A machine is a system of a more complex order; displace a lever, unscrew a bolt, re-distribute the parts, and the machine as a machine ceases to exist. The body of a living creature is a system of a still more complex order.

If then we suppose some knowledge of a system, the possibility of inference is at once apparent. "Thus consider the case of a workman who, when in the course of excavation, turns up a human jawbone. He may not recognise it as such, or, if he does, he regards it curiously and casts it away. He has no useful system into which the fact will fit, and so receive a fuller meaning. But the geologist with the same preliminary data can commence a process of thought by which he constructs mentally the conditions of life which must have prevailed during the time when the human being of whom this sole relic remains was still alive. He decides perhaps that it must have been preserved from pre-historic times. The anatomist, too, from the size and shape of the bone, can tell a great deal about the size and shape of the physical frame with which

alone it would have been consistent. He may, perhaps, show that only muscles of much greater strength than is common to-day could have moved it. In some such way, a number of inferences impossible to the workman might be drawn by the man of science because he possesses a knowledge of the *systems* appropriate to the fact to be interpreted. But he, too, would have failed to make an inference if he had not had a system of inter-related truths enabling him to pass *from* what was given, and recognised as part of the system, *to* its meaning in the light of the whole,"[1] and (we may add) enabling him to pass back again *from* that "meaning" *to* further understanding of the given fact. He reasons "from particular to general" and back "from general to particular."

The variety of propositional forms, and their possible complexity, as illustrated in Chapter IV, suggests also how various and complex the forms of deductive inference may be.

The student of mathematics cannot fail to observe that the whole of Pure Mathematics is a deductive development of a distinctive system. Laying down a limited number of first principles, self-evident or to be accepted as postulates, it evolves a vast number of deductive truths by a procedure in the highest degree systematic. The rigid definition of leading terms and conceptions; the explicit statement of all the first principles, the onward march by successive deductions, each one resting on ground already secured; no begging of either premisses or conclusions, no surreptitious admissions; no shifting of ground; no vacillation in the meanings of terms—all this is implied in the perfect type of a deductive science Such is, to a very considerable degree, the impression made by a course of mathematical instruction. It would be made in a still higher degree if this science had been more true to itself and had not permitted a certain looseness in the treatment of the first principles, of which there are some conspicuous examples in some of the "Definitions" and "Axioms" of Euclidean geometry. These defects are now being remedied.

On the meaning of "system," see Welton and Monahan, *An Intermediate Logic* (third edition, revised by E. M. Whetnall),

[1] Welton and Monahan, *Intermediate Logic*, revised by E. M. Whetnall, Chapter XXI., § 6.

Chapter XXI., § 5 and 6; and Chapter XXXIII., § 1 to 3; and for a more detailed statement, Stebbing, *A Modern Introduction to Logic*, Chapter XIII., § 1; Chapter XI.; and Chapter X., § 4 (on the meaning of a "deductive system").

2. The Syllogism: The First Figure

We shall explain deductive reasoning in its simplest form. The simplest kind of deduction is that in which our "system" takes the form of a general rule or principle with which we start; and the reasoning consists in combining a related fact with the general rule through some characteristic which actually connects it with the rule. We shall use only two-term propositions expressed in one or other of the forms **A, I, E, O**; and the type of reasoning which we shall explain and discuss is that by which from two given propositions (at least one of them being universal) we pass to a third proposition whose truth necessarily follows from the truth of these given propositions.

An argument of this type is called a **Syllogism**, which means "a thinking together," *i.e.* thinking two propositions together. The two propositions must not be equivalent; so that neither of them can be derived by Immediate Inference from the other. The two given propositions are called the **premisses**, and the third proposition, derived from the two premisses together, and not derivable from either of them taken separately, is called the "conclusion." It is evident that we do not necessarily derive a conclusion from any pair of propositions whatever, as for example, from "all men are fallible" and "all metals are elements." These statements have nothing in common. In order that two propositions may result in a conclusion they must have something in common; and this means that when expressed in logical form they must have a common term, otherwise there is no link of connection between them.

We have said that syllogistic reasoning is essentially the connecting of a "general rule" and a "related fact." The "related fact" may itself be a universal proposition which, for the purposes of the given argument, comes under the "rule"; or it may be a particular case or detail. Bearing this in mind, we have now to see that a general rule and a related fact may be connected in more than one way. We shall consider *first* the extremely important case in which we are able to conclude something about the related fact because it comes under the general rule *as an example or application of it.* As we have seen, the general rule may be stated either as a universal affirmative or as a universal negative proposition.

(1) Let us consider examples of reasoning in which the general rule is stated in the form of a universal affirmative proposition. The following is an example:—

$$\begin{cases} \textit{All men are fallible,} \\ \textit{All philosophers are men.} \end{cases}$$

Here the "general rule" is expressed as a universal proposition affirming a certain quality of "all men." The "particular fact," though here it also is a universal proposition, comes under the general rule as an application of it; and the conclusion is:

All philosophers are fallible.

Or, again, take the following example:—

$$\begin{cases} \textit{All men are fallible,} \\ \textit{Aristotle is a man.} \end{cases}$$

Here the same quality is affirmed of "all men," and the particular case is the individual Aristotle—a Singular term (Proper Name), though as we have seen (page 61) the

proposition ranks technically as a universal; and the conclusion is:—

Aristotle is fallible.

The typical example of this first kind of deductive syllogistic reasoning is therefore as follows:—

$$\textit{Premisses} \begin{cases} \text{All M is P} \\ \text{All S is M} \end{cases}$$

$$\textit{Conclusion} \ \text{All S is P.}$$

We use "M" as a symbol of the common term which links the two propositions together, and which for that reason is called the **middle term.** And the two terms which appear in the conclusion also require technical names: the subject-term of the conclusion is called the **Minor Term,** and the predicate of the conclusion is called the **Major Term.** The premiss which contains the major term is called the **Major Premiss,** and the premiss which contains the minor term is called the **Minor Premiss.** In the kind of syllogism which we are now considering, and which has been illustrated above, the major premiss is the one which states the general rule, and the minor premiss is the one which brings the related fact under the general rule. In the first example, the major premiss is "All men are fallible"; the minor premiss is "All philosophers are men," and the conclusion is "All philosophers are fallible." The major term (predicate of the conclusion) is "fallible"; the minor term (subject of the conclusion) is "philosophers," and the middle term (occurring in both premisses) is "men." In the second example, the major premiss is again "All men are fallible"; but the minor premiss is "Aristotle is a man"; and the conclusion is "Aristotle is fallible." The middle term is "men"; the major term

"fallible," and the minor term "Aristotle." These definitions may be thus expressed symbolically:

All M is P

(*major premiss, containing middle and major terms*);

All S is M

(*minor premiss, containing minor and middle terms*);

All S is P

(*conclusion, containing minor and major terms*).

All syllogisms in which the middle term is subject of the major premiss and predicate of the minor premiss are said to be of the **First Figure.** The reason for this technical term will be made clear later on. The form of the First Figure which we have illustrated may conveniently be denoted by the symbol **AAA**, from the form of its constituent propositions. In the first Figure the major premiss not only contains the major term but also states the "general rule" on which the argument depends, and must therefore be a universal proposition; and the minor premiss, which brings forward the "related fact," must be affirmative. If the minor premiss were negative, it would simply say that the particular case does not really come under the general rule, and there would be no conclusion. Let us take an example:—

All men are mortal,
No angels are men.

Here the minor premiss simply *excludes* the class "angels" from the general rule stated in the major premiss; and the mere *exclusion* of an apparently related fact from the general rule does not entitle us to draw a conclusion. Thus from the premisses which we have just given, we cannot conclude—

No angels are mortal;

even though the apparent conclusion *may as a fact* be true. For another similar case take the following:—

All graduates may vote in the University Election ;
No negroes are graduates.

From these premisses we cannot draw the conclusion—

No negroes may vote,

although this statement is true as a matter of fact. These examples show the importance of distinguishing between (i.) a conclusion which is *valid,* that is, which follows logically from the premisses, and (ii.) a conclusion which, though true, does not follow logically from the premisses.

(2) In the examples already given, in the first Figure, the minor premiss has been a universal affirmative proposition as well as the major. Now suppose that the minor premiss is not a universal affirmative but a particular affirmative proposition. The form then becomes:—

All M is P,
Some S is M;

and here the minor says nothing about "All S"; it only affirms that some at least of S comes under the rule stated in the major. Hence we cannot say anything about "All S" in the conclusion; we can only conclude:—

Some S is P.

This is the form **AII** in the first Figure. It is merely a more indefinite form of the process of reasoning employed in **AAA.** The following are examples illustrating its use. "Every act which is done from a strict sense of duty is subjectively right; some acts which mankind generally condemn are done from a strict sense of duty; therefore some acts which are generally condemned are subjectively right." And again: "All governments which aim at

securing the welfare of their people are just governments; some dictatorships have aimed at securing the welfare of the people; therefore some dictatorships have been just governments."

The conclusion, though indefinite, excludes its contradictory, "No acts which are generally condemned can be in any sense right," which is a debated proposition and an important one. And in like manner, in the other example, the conclusion excludes the proposition that "No dictatorship can possibly be just," which again is a debated opinion.

(3) In the previous examples of syllogisms in the first Figure, the major premisses have been universal affirmatives. The general rule, however, may take the form of a universal negative; and then the form becomes:—

No M is P,
All S is M;

here the minor states that all of S comes under a class wholly excluded from P; and we can only conclude:—

No S is P.

This is the form **EAE** in the first Figure. It is of less importance than the form **AAA**; we can only clear the ground, not directly advance knowledge, by proving what a thing *is not*. Thus we may argue: "Nothing involuntary can be cured by punishment; all stupidity is involuntary; therefore no stupidity can be cured by punishment." Assuming the minor premiss to be true, the argument tells us nothing of how stupidity *may* be cured.

We may compare the valid form **EAE** in the first Figure with the invalid **AEE** to which reference has already been made (page 110). In the case of **AEE** the minor premiss simply *excludes* the class denoted by the minor term S

from the general rule stated in the major premiss. Hence, as we have seen, no valid conclusion can be drawn from the premisses **AE** in the first Figure. But the case of **EAE** in the same Figure is entirely different. The minor premiss makes an affirmative statement about the class denoted by the minor term: namely, that it comes under the class M which, the major premiss tells us, is wholly excluded from P. Hence we have the valid conclusion " No S is P."

(4) Finally, when the major premiss is a universal negative proposition, and the minor premiss not a universal affirmative (as in **EAE**) but a particular affirmative, we have the form **EIO**, which holds the same relation to **EAE** as **AII** does to **AAA**:—

No M is P,
Some S is M;

from which the only possible conclusion is the particular negative—

Some S is not P.

We have found, then, that there are four forms of syllogism possible in the first Figure:—

AAA, EAE, AII, EIO.

We are using only the four propositional forms recognised in the traditional classification of categorical propositions; hence no other forms of syllogism are possible in the first Figure; which from its very nature requires a universal major premiss and an affirmative minor (see page 110). These four forms are known by the technical names—

Barbara, Celarent, Darii, Ferio.

These names contain the vowel-symbols of the respective forms in the proper order, the major premiss being placed

first. The reader will see shortly that this by no means exhausts the utility of these peculiar words.

The first Figure is of the greatest importance both in science and in common life. Whenever we apply previous knowledge to a particular case we employ one of the forms of this Figure—although no formal syllogism may be before our minds. The form **AAA** is so familiar and constant a mode of thought that its importance usually escapes notice. But we might know beforehand the large part it must play in science; for science seeks for results which are *laws*—that is, statements true universally about certain kinds of fact. Every time we explain a fact by a law, in other words, every time we make a new application or find a new exemplification of the law, we make a syllogism of the form **AAA** (or, occasionally, **AII**) in the first Figure—not explicitly or in expression, for this would make the reasoning long and tedious; but implicitly at every step we reason in this form.

The following are simple examples of this kind of reasoning, in which the major premiss states a scientific law (see also Chapter I., page 14): "All material bodies which are heated expand in bulk and volume; this pendulum is a material body heated; hence it expands in volume." And again: "A pendulum lengthened is a pendulum which swings in a longer path; the heated pendulum is lengthened; therefore the path in which it swings is lengthened." These conclusions show the bearing of changes of temperature on the regularity of a clock. In all such reasoning, the major premiss (the general law) is supposed to be known independently of the case to which we apply it. In the following example Newton formulated the major premiss on the basis of mathematical calculation: "Any body revolving round

another which attracts it with a force decreasing as the square of the distance increases, is a body which will describe an orbit of which Kepler's laws are true; the planets are bodies holding this relation to the sun; therefore the planets describe orbits of which Kepler's laws are true." And Kepler's laws were based upon an enormous number of astronomical observations.

In *Grammar*, every application of a rule to the construction of a sentence is a syllogism in *Barbara*. In *Ethics*, all appeals to accepted moral rules in judging particular acts are syllogisms in Figure I., and if the conclusion is affirmative, the form is the same fundamental one In *Law*, the procedure is equally syllogistic. "The whole aim of legal procedure is to determine whether a particular case does or does not fall under a given general rule. Thus, in a criminal trial, the law which has been violated furnishes the major premiss, and the examination of the acts of the accused supplies the minor premiss." In *Economics*, the whole "Deductive Method" is an application of general rules to cases coming under them, and therefore consists in a continual use of the form *Barbara*. In *History*, explanation by general laws is resorted to whenever possible. Our knowledge of human nature, individual and social, supplies various major premisses, of what men and nations will do under given circumstances; and having found historical examples of these circumstances, we explain what occurred by reasonings in *Barbara*. In *Medicine*, any case of *diagnosis* is a syllogism in this form. Certain bodily conditions are known to be the symptoms of a certain disease; this is a case of those conditions; therefore this is a case of that disease.

For an elementary account of the Aristotelian view of the first Figure, see Mellone, *Introductory Textbook of Logic*, Chapter VI, § 2, 6, and 10, and Chapter VII, "Note B" (page 237); and for a fuller statement, see Joseph, *Introduction to Logic*, second edition, Chapters XIII. and XIV. For a very full and sound account of the meaning of the syllogistic Figures, with numerous examples, see Ueberweg, *Logic*, Eng. trans., pages 404 and following.

We must add that it is logically indifferent whether the major premiss or the minor stands first. The two following syllogisms are logically the same:—

(1) M *a* P,	(2) S *a* M,
S *a* M;	M *a* P;
∴ S *a* P.	∴ S *a* P.

It is, however, an almost invariable custom to place the major premiss first, as in (1), and each of the previous examples. We shall therefore adopt the following definitions: *the premiss which contains the major term is called the major premiss, and the premiss which contains the minor term is called the minor premiss.* These definitions are applicable not only to the first Figure but also the other forms of syllogistic reasoning which we now proceed to discuss.

3. The Syllogism: Its Remaining Figures

We observed above that a general rule and a related fact may be connected in more than one way, giving rise to more than one type of syllogistic reasoning. We have explained and discussed one important type of such inference, in which the major premiss states a general rule or principle, and the minor brings forward a related fact to which it applies.

In the next place we have to examine the case in which the fact and the general rule stand in a relation of mutual opposition or exclusion. The related fact *has* a characteristic which is excluded from everything coming under the general rule; or it *has not* a characteristic which everything coming under the general rule must have. The common term or middle term stands for this characteristic which belongs to one and is excluded from the other; so that it is predicate in both premisses. The general rule is stated in the major premiss which is therefore universal; and from what we have just said it follows that if the major premiss is affirmative, the minor must be negative, and if the major is negative, the minor must be affirmative. The conclusion is necessarily negative. Hence it has been said that syllogisms of this type establish distinctions

between things. We prove a distinction between S and P by pointing out that P has a characteristic M which S has not, or that P has not a characteristic M which S has. The two following forms arise under these conditions:—

(1)	All P is M;	(2)	No P is M;
	No S is M;		All S is M;
∴	No S is P.	∴	No S is P.

Thus, suppose that we wish to establish a distinction between "Christmas roses" and true roses. For the middle term we look for a characteristic which belongs to the one and not to the other:—

All true roses bloom in summer,
No Christmas roses bloom in summer;
Therefore no Christmas roses are true roses.

This is a syllogism of the form (1) above. Suppose, again, that we wish to establish a distinction between whales and fishes: this may be done in a syllogism of the form (2) above:—

No fishes breathe through lungs;
All whales breathe through lungs;
Therefore no whales are fishes.

Syllogisms of this type in which the middle term is predicate in both premisses, are said to be of the **Second Figure.**

The minor premiss need not be a universal proposition. But if the subject becomes indefinite through the use of "some at least," the conclusion must be equally indefinite.

This gives two more forms, corresponding respectively to those marked (1) and (2) above:—

(3)	All P is M;	(4)	No P is M;
	Some S is not M;		Some S is M;
∴	Some S is not P.	∴	Some S is not P.

The following are examples of the form (3): "All true theories are self-consistent; Some scientific theories are not self-consistent; Therefore some scientific theories are not true." Or, again: "All moral acts are done from a praiseworthy motive; some legal acts are not done from a praiseworthy motive; therefore some acts that are legal are not moral." And the following are examples of the form (4) above: "No fishes have lungs; some aquatic animals have lungs; therefore some aquatic animals are not fishes" (it will be seen that this is based on or derived from the argument about "whales" which we have quoted). Or, again, "No forces which work by strictly mechanical laws can produce organic beings capable of growth and reproduction; some forces in Nature have produced such things; therefore some forces in Nature are not forces working by strictly mechanical laws."

Thus, in the second Figure we have four forms:—

EAE, AEE, EIO, AOO.

These forms are technically known by the following names:

Cesare, *Camestres*, *Festino*, *Baroco*.

We shall point out the meanings of these words shortly.

Finally we will examine the case in which we desire to prove that two characteristics are *compatible* (capable of being possessed by the same subject) or *incompatible*, as the case may be. As we have seen (page 66) compatibility

is logically expressed by the particular affirmative proposition, and incompatibility by the particular negative. We prove that "some S is P" because we can point to a class which is both S and P; and that "Some S is not P" because we can point to a class which is at once S and not P. The *same class* must be referred to in both premisses, so that the term denoting it must be the *middle term*; and since we are appealing to characteristics which it possesses, it must be Subject in both premisses. The *minor* premiss must be affirmative, since it connects the characteristics (whose compatibility or incompatibility we desire to prove) with the class denoted by the middle term; and the conclusion must be particular (**I** or **O**). Here the "general rule" is not, as in the first Figure, applied to a case which may be characterised by it; it is treated as a class of examples of the conclusion which we desire to prove. In this sense, the principle of this type of syllogism is the reverse of the principle of the first Figure.

The following forms are possible under these conditions:—

AAI, IAI, AII, EAO, OAO, EIO.

These forms are technically known by the following names:

Darapti, Disamis, Datisi, Felapton, Bocardo, Ferison.

Syllogisms of this type, in which the middle term is subject in both premisses, are said to be of the **Third Figure.**

These forms are not all independent of one another. The form **EAO** is the negative corresponding to the affirmative **AAI**, and these two are of course independent; but **IAI** and **AII** may be derived from **AAI**, and in like manner **OAO** and **EIO** may be derived from **EAO**. By substituting for the major premiss of **AAI** its "subaltern"

we get **IAI**, with the same conclusion; and by substituting for the minor its subaltern, we get **AII**:—

(1) All M is P,
All M is S;
∴ Some S is P.

(2) Some M is P,
All M is S;
∴ Some S is P.

(3) All M is P,
Some M is S;
∴ Some S is P.

Similarly, by substituting for the major premiss of **EAO** its subaltern, we get **OAO**, with the same conclusion; and by substituting for the minor its subaltern, we get **EIO**:—

(4) No M is P,
All M is S;
∴ Some S is not P.

(5) Some M is not P,
All M is S;
∴ Some S is not P.

(6) No M is P,
Some M is S;
∴ Some S is not P.

The form **AAI** in the third Figure contains more than enough to warrant its "particular" conclusion; and the same is true of the form **EAO**. The argument in each case is perfectly valid; but the middle term is distributed twice in each case. The following are examples:—

(1) *All whales are mammals;*
All whales are aquatic animals;
Therefore some aquatic animals are mammals.

(2) *No ostrich is able to fly;*
All ostriches have wings;
Therefore some winged animals are not able to fly.

The form **AAI** is specially appropriate when the middle is a Singular term. In this connection we must again emphasise the fact that a proposition making an assertion about a Singular subject is ranked technically as a universal proposition (**A** or **E**). If, for example, we require

an instance in support of the conclusion that artistic genius and scientific ability are compatible, we may argue:—

Leonardo was a man of scientific ability;
Leonardo was a man of artistic genius;
Therefore some men of artistic genius are men of scientific ability.

As a matter of fact, this form of the third Figure (**AAI** with the middle term Singular) shows most clearly what the nature of the reasoning in this Figure really is. The common factor is an individual instance, as in this case, or a class of instances, on which is based a conclusion in the form " Some (at least) . . ." What we cannot do, in the third Figure, is to base a *universal* conclusion on the instances; we can never say:—

This M is P,
This M is S;
∴ All S is P;

and we can never say:—

All M is P,
All M is S;
∴ All S is P.

But the " particular " conclusion, which is the valid one, *suggests* that the possibility of the universal conclusion may be made a subject of further inquiry (see a simple example of this in Chapter I., page 10).

We have now explained and examined three types of syllogistic reasoning. We have seen that they are three independent types. Each has, so to speak, a distinctive " nature " of its own, formally expressed in the structure of the syllogisms which typify its various forms, and

particularly in the position of the middle term, on which the technical distinction of three figures is based.

We have also seen that from examination of the nature of each of these types of reasoning, special rules may be derived regulating the structure of the syllogisms which express its various forms. We have formulated these rules in the course of the foregoing discussion. For convenience we now re-state them separately:—

FIGURE 1

1. The major premiss must be universal.
2. The minor premiss must be affirmative.

FIGURE 2

1. The major premiss must be universal.
2. One premiss must be negative.
3. The conclusion must be negative.

FIGURE 3

1. The minor premiss must be affirmative.
2. The conclusion must be particular.

A convenient general name for the valid forms of syllogism in any Figure is in use. They are called the **moods** of that Figure. Thus the valid "moods" of the first Figure are **AAA, EAE, AII, EIO.** We have seen how in each Figure the valid mood may be determined by special rules derived from the principle of the Figure. The *mood* of a syllogism depends on the quality (affirmative or negative) and quantity (universal or particular) of its premisses; whereas its *figure* depends on the positions of its middle term in its premisses. Now if we look at the matter from the point of view of mechanical symmetry, we find that there is a fourth possible type of syllogism.

We have already three types: (1) the middle term subject of the major premiss and predicate of the minor; (2) the middle term predicate of both premisses; (3) the middle term subject of both premisses. Hence it is natural to inquire whether there may be a case (4) in which the middle term is predicate of the major premiss and subject of the minor. As a matter of fact, Aristotle recognised the possibility of syllogistic arguments of this type, but he regarded them only as a variety of the first Figure. Later Greek logicians worked out the possible forms and made them into a separate "fourth Figure," defined by the position of the middle term—predicate of the major premiss and subject of the minor. There are five possible forms of syllogisms in the fourth Figure:—

AAI, AEE, IAI, EAO, EIO.

These forms are known by the following technical names:—

Bramantip, Camenes, Dimaris, Fesapo, Fresison.

The awkwardness of the fourth Figure is due to the fact that a term which is naturally predicate is taken as subject in the conclusion. Thus, if we have these premisses—

All roses are plants,
All plants need air,

we should naturally expect the conclusion to be about "roses"—that is, we should naturally regard the syllogism as one in *Barbara,* first Figure, the conclusion being—

All roses need air.

But in the fourth Figure the conclusion unexpectedly makes the statement about "things needing air"—

All roses are plants,
All plants need. air,
Therefore some things needing air are roses.

This is the mood **AAI** in the fourth Figure. It is entirely superfluous, as well as unnatural, for the conclusion, if desired, can be obtained by merely converting the conclusion of *Barbara*. The same remark applies to the forms **AEE** and **IAI** in the fourth Figure, in which the conclusion, when we think naturally, is drawn in *Celarent* and *Darii* respectively; and if the conclusion of the fourth Figure is required, it is obtained by conversion.

The two remaining moods of the fourth Figure, **EAO** and **EIO**, fall less readily into the form of the first Figure. If we convert both premisses of **EAO**, remembering that the proposition **A** converts to **I** (see page 89), we get a pair of premisses from which the conclusion of **EAO** (in the fourth Figure) follows in *Ferio* in the first; also **EIO** may be derived from **EAO** by taking the subaltern of the major premiss in the latter But **EAO** and **EIO** (fourth Figure), by simple conversion of the major premiss in each, are transformed into *Felapton* and *Ferison* (*third* Figure). For full discussion of the history and value of the fourth Figure, see Joseph, *Introduction to Logic*, second edition, pages 280-85, 325-30

Examination of the five moods of the fourth Figure shows that they exemplify the following rules, which may therefore be called the Special Rules of this Figure —

(1) If the major premiss is affirmative, the minor must be universal.

(2) If the minor premiss is affirmative, the conclusion must be particular.

(3) If either premiss is negative, the major premiss must be universal.

4. Rules of the Syllogism

Long after the valid moods of the syllogistic Figures had been discovered and analysed, the general principles common to all these valid moods were set out in a code of rules known as the " General Rules of the Syllogism " (to distinguish them from the Special Rules peculiar to each Figure). The examination and illustration of the rules is a valuable exercise in clearness and distinctness of thought and expression, and throws light on many confusions

which occur in ordinary reasonings. We shall employ the *class-interpretation* of propositions, as we did in the examination and illustration of the rules of Immediate Inference. That is to say: we shall understand the preposition **A** as stating that " All of the class S falls within the class P "; the proposition **I**, that " some (at least) of the class S falls within the class P "; the proposition **E**, that " None of the class S falls within the class P "; the proposition **O**, that " Some (at least) of the class S falls outside the class P." We use the expression " falls within " as equivalent to " is included in," and " falls outside " as equivalent to " is excluded from."

The most complete version of the " code " gives eight rules.

I. Relating to the structure of the syllogism:—

(1) A syllogism must contain three, and only three, propositions.

(2) A syllogism must contain three, and only three, terms.

II. Relating to *quantity*:—

(3) The middle term must be distributed in one, at least, of the premisses.

(4) No term must be distributed in the conclusion unless it is distributed in the premiss which contains it.

III. Relating to *quality*:—

(5) From two negative premisses there can be no conclusion. In other words: One, at least, of the premisses must be affirmative.

(6) If one premiss is negative, the conclusion must be negative, and if the conclusion is negative, one premiss must be so.

IV. *Corollaries*:—

(7) From two particular premisses, there can be no conclusion.

(8) If one premiss be particular, the conclusion must be particular.

The *first and second* rules tell us what a syllogism is. We have seen that there must be two premisses and a conclusion, and that the premisses must have a common term; and, since both the constituent terms of the conclusion occur also as constituents of the premisses, there can only be three terms. When rule 2 is violated it is usually through ambiguities of language. An *ambiguous term* is not *one* term but two (or even more); hence the syllogism containing an ambiguous term has at least four terms. When there is ambiguity, it is most likely to be in the middle term; **ambiguous middle** is the most common breach of rule 2.

Some interesting examples are given by Jevons. If we argue that "all metals are elements, and brass is a metal, therefore it is an element," we should be using the middle term *metal* in two different senses, in one of which it means the pure simple substances known to chemists as "metals," and in the other a mixture of metals commonly called "metal" in the arts, but known to chemists by the name "alloy." In many examples which may be found in books on Logic the ambiguity of the middle term is very obvious, but the reader should always be prepared to meet with cases in which exceedingly subtle and difficult ambiguities occur. Thus it might be argued that "what is right should be enforced by law, and that charity is right and should therefore be enforced by law." Here it is evident that "right" is applied in the latter case to what the

conscience approves, and in the former case to what public opinion holds to be necessary for the good of society. Different opinions may be held concerning some of these ambiguities. Thus we might argue: "He who harms another should be punished; he who communicates an infectious disease to another person harms him; therefore he who communicates an infectious disease to another person should be punished." This may or may not be held to be a correct argument, according to the kinds of actions which we should consider to come under the term "harm." Many difficult questions are of this nature, as for instance: "Nuisances are punishable by law; to keep a noisy dog is a nuisance; hence to keep a noisy dog is punishable by law." Here the question turns upon the degree of nuisance which the law would interfere to prevent. Or, again: "Interference with another man's business is illegal; underselling interferes with another man's business: hence underselling is illegal." Here the question turns upon the *kind of interference*, and it is obvious that underselling is not the kind of interference referred to in the major premiss.

The examples of "ambiguous middle" are instances of a kind of Fallacy which has been called "Equivocation" (on the meaning of the term "Fallacy," see Chapter II., page 29). Equivocation, as a logical fallacy, is a general name for errors in reasoning, due to the *use of the same term in different senses in the course of the same argument.* In this general sense, Equivocation is a violation of the Law of Identity in reference to the meanings of terms.

We add some further examples of ambiguous middle, which the reader may examine for himself. (1) "*Finis rei est illius perfectio; mors est finis vitae; ergo, mors est perfectio vitae*" ("the end of a thing is the perfection of it; death is the end of life; therefore death is the perfection

of life "). (2) " All criminal actions ought to be punished by law; prosecutions for theft are criminal actions, and therefore ought to be punished by law." (3) " Every good law should be obeyed; the law of gravitation is a good law, and therefore should be obeyed." (4) " Partisans are not to be trusted; the supporters of the Government are partisans, and therefore are not to be trusted."

The *third* rule states that the whole extent of the middle term must be referred to universally in at least one of the premisses. Otherwise, we might be referring to one part of it in one premiss and another part of it in the other premiss; in such a case there would be no real middle term at all—no term common to the two premisses.

In this connection it is interesting to observe the connection between the " undistributed middle " and reasoning from Analogy. Analogy is " any resemblance between things which enables us to believe of one what we know of the other." The value of the inference depends on whether the resemblance is in the material or essential points; but it is never completely conclusive. Now an argument from Analogy, if put into syllogistic form, becomes an attempt to derive an affirmative conclusion from two affirmative premisses in the second Figure:—

All P is M,
All S is M;
∴ All S is P;

where M in each case is the undistributed predicate of an affirmative proposition, and, formally, no conclusion follows. All that such an argument can do is to suggest the *possibility* of a real connection between S and P. For a simple example, see Chapter I., page 11; and for further discussion of Analogy, Chapter VII., Section 4.

The *fourth* is a double rule. (*a*) The minor term must not be distributed in the conclusion unless it is distributed in the premiss in which it occurs; the breach of this rule is technically termed " illicit process of the minor." (*b*) The major term must not be distributed in the conclusion unless it is distributed in the premiss in which it

occurs; and the breach of this rule is technically termed "illicit process of the major." An "illicit process of the minor" is generally easy to detect; in the case of the major, it may be much less apparent. The following, for example, might pass for a correct syllogism, especially as the conclusion may be accepted as true: "All rational beings are responsible for their actions; the lower animals are not rational beings; therefore the lower animals are not responsible for their actions." Here the form is:—

All M is P,
No S is M;
∴ No S is P.

The major term P—"beings responsible for their actions"—is distributed in the conclusion but is not distributed when it stands as the predicate of an **A** proposition in the major premiss.

We must observe that this rule, though it forbids us to take *more* of a class in the conclusion than was referred to in the premiss, does not forbid us to take *less*. There is no "illicit process" when a term is *distributed in the premiss* and undistributed in the conclusion, as in the following:—

All M is P,
All S is M;
∴ Some S is P.

It may be considered a logical fault to take less in the conclusion than is warranted by the premisses; but it is not technically a breach of any syllogistic rule. In fact, we must observe that this so-called "logical fault" is often simply a matter of the *relevance* of the conclusion, when the "particular" conclusion may be more useful, for the

purpose in hand, than the "universal" one. For example:—

All University students are eligible for the Prize;
All members of this College rank as University students;
Therefore the whole of this class (i.e. *some members of this College*) *may enter for the Prize.*

Or, again, from the same premisses:—

First-year students (i.e. *some who rank as University students*) *are entitled to enter.*

This might be a retort to the suggestion that they are not eligible.

The *fifth* rule states that one premiss, at least, must be affirmative, or, which is the same thing in other words, from two negative premisses there can be no conclusion. A negative major premiss means the denial of any connection between the major term and the middle. A negative minor premiss means the denial of any connection between the minor term and the middle. The condition of a valid syllogism does not exist; there is really no middle term at all.

Jevons, in his *Elementary Lessons in Logic,* gave the following explanation of the case—not of uncommon occurrence—where from two apparently negative premisses we obtain a valid conclusion. The mere occurrence of a negative particle ("not" or "no") in a proposition does not render it negative in the manner contemplated by this rule. Thus the argument—

"What is not compound is an element,
Gold is not compound,
Therefore gold is an element,"

contains negatives in both premisses, but is nevertheless valid, because the negative in both cases affects the middle term, which is really the negative term "non-compound." Now this explanation applies to an example which Jevons himself gives, in his *Principles of Science,* as a case where two negative premisses give a valid conclusion. The example is—

"Whatever is not metallic is not capable of powerful magnetic influence,

now to show that all the valid Moods can be determined from the rules which we have just explained.

The Mood of a syllogism depends on the quantity and quality of its premisses. There are two premisses, and each premiss must be **A**, **I**, **E**, or **O**. Hence the greatest possible number of Moods (valid or not) will be the number of permutations of these four letters, two at a time. There are in all sixteen such permutations:—

AA	**IA**	**EA**	**OA**
AI	**II**	**EI**	**OI**
AE	**IE**	**EE**	**OE**
AO	**IO**	**EO**	**OO**

In this table **AA** means, of course, that both premisses are universal affirmative; **IA**, that the major is particular affirmative, the minor universal affirmative; and so on. In each case the first of the two letters denotes the major premiss, and the second the minor premiss.

Of these sixteen moods, nine are excluded by the general rules: **EE**, **EO**, **OE**, **OO**, by the rule against two negatives; **IO**, **II**, **OI** by the rule against two particulars. And **IE** can be shown to yield no conclusion in any figure; thus: "If possible, let there be a conclusion: then it must be negative. Therefore its predicate (the major term) is distributed; but the major premiss (**I**) distributes neither subject nor predicate. Therefore the conclusion would involve an illicit major." We are thus left with eight combinations of premisses: **AA**, **AE**, **AI**, **AO**, **EA**, **EI**, **IA**, **OA**. We have to determine which of these are valid in each of the four Figures, by reference directly to the general rules.

(1) In the first Figure, where the middle term is subject in the major premiss and predicate in the minor, it will be found that **AA**, **EA**, **AI**, and **EI** yield conclusions valid

under all the rules, giving the same four forms which we have already derived from first principles, and which are known by the technical names *Barbara, Celarent, Darii, Ferio.* Of the four remaining combinations of premisses the reader will see on examination that **AE** and **AO**, both requiring negative conclusions, involve in each case an illicit major:—

$$\begin{cases} M\ a\ P, \\ S\ e\ M, \end{cases} \qquad \begin{cases} M\ a\ P, \\ S\ o\ M, \end{cases}$$
$$S\ e\ P \text{ or } S\ o\ P; \qquad S\ o\ P;$$

and that **IA** and **OA** both involve undistributed middle:—

$$\begin{cases} M\ i\ P, \\ S\ a\ M; \end{cases} \qquad \begin{cases} M\ o\ P, \\ S\ a\ M. \end{cases}$$

(2) In the second Figure, when M is predicate in both premisses, it will be found that **EA**, **AE**, **EI**, and **AO** yield conclusions valid under all the rules, giving the forms known technically as *Cesare, Camestres, Festino, Baroco.* Of the four remaining combinations of premisses, the reader will see on examination that **AA**, **AI**, and **IA** each involve an undistributed middle:—

$$\begin{cases} \text{All P is M,} \\ \text{All S is M;} \end{cases} \qquad \begin{cases} \text{All P is M,} \\ \text{Some S is M;} \end{cases} \qquad \begin{cases} \text{Some P is M,} \\ \text{All S is M;} \end{cases}$$

and that **OA**, requiring a particular negative conclusion, involves illicit major.

(3) In the third Figure, where the middle term is subject in both premisses, it will be found that **AA**, **IA**, **AI**, **EA**, **OA**, **EI**, yield conclusions valid under all the rules, giving the forms known technically as *Darapti, Disamis, Datisi, Felapton, Bocardo, Ferison.* The two remaining

combinations of premisses, **AE** and **AO**, both requiring negative conclusions, involve in each case an illicit major:—

$$\begin{array}{ll} \left\{\begin{array}{l}\mathrm{M}\ a\ \mathrm{P},\\ \mathrm{M}\ e\ \mathrm{S},\end{array}\right. & \left\{\begin{array}{l}\mathrm{M}\ a\ \mathrm{P},\\ \mathrm{M}\ o\ \mathrm{S},\end{array}\right. \\ \quad \mathrm{S}\ e\ \mathrm{P} \text{ or } \mathrm{S}\ o\ \mathrm{P}; & \quad \mathrm{S}\ o\ \mathrm{P}. \end{array}$$

(4) In the fourth Figure, where the middle term is predicate in the major premiss and subject in the minor, it will be found that **AA**, **AE**, **IA**, **EA**, and **EI** yield conclusions valid under all the rules, giving the forms known technically as *Bramantip*, *Camenes*, *Dimaris*, *Fesapo*, *Fresison.* There are three remaining combinations of premisses: of these, the reader will see that **AO** and **AI** involve undistributed middle, and that **OA** involves illicit major:—

$$\begin{array}{lll} \left\{\begin{array}{l}\mathrm{P}\ a\ \mathrm{M},\\ \mathrm{M}\ o\ \mathrm{S},\end{array}\right. & \left\{\begin{array}{l}\mathrm{P}\ o\ \mathrm{M},\\ \mathrm{M}\ a\ \mathrm{S},\end{array}\right. & \left\{\begin{array}{l}\mathrm{P}\ a\ \mathrm{M},\\ \mathrm{M}\ i\ \mathrm{S},\end{array}\right. \\ \quad \mathrm{S}\ o\ \mathrm{P}; & \quad \mathrm{S}\ o\ \mathrm{P}; & \quad \mathrm{S}\ i\ \mathrm{P}. \end{array}$$

It remains to show that the "special rules" of each of the Figures can be directly derived from the general rules. We will work out these results in full, in the case of the first, second, and third Figures, in order to illustrate the method of treating deductions from the rules.

Figure I

Rule i.—*The minor premiss must be affirmative.*

If possible let the minor premiss be negative. Then the major must be affirmative, and P, its predicate, undistributed. Also the conclusion must be negative, and P, its predicate, distributed. Hence if the minor premiss is negative, we have an illicit major. Therefore the minor must be affirmative.

RULE 2.—*The major premiss must be universal.*

The minor premise must be affirmative, and the middle term, its predicate, undistributed. Therefore the middle term must be distributed in the major premiss, where it is subject; hence the major premiss must be universal.

FIGURE II

RULE 1.—*One of the premisses must be negative.*

The middle term must be distributed once, at least, in the premisses; and, as it is predicate in both premisses, its distribution can only be secured when one of them is negative.

RULE 2.—*The major premiss must be universal.*

As one premiss is negative, the *conclusion* must be negative, and the major term P, its predicate, distributed; P must therefore be distributed in the major premiss, of which it is subject; that is, the major must be universal.

FIGURE III

RULE 1.—*The minor premiss must be affirmative.*

The proof is exactly the same as in the case of Rule 1 of the first Figure. The proof turns on the fact that P, the major term, is predicate in the major premiss as well as in the conclusion: and this is true both in Figure I and Figure III.

RULE 2.—*The conclusion must be particular.*

The minor premiss is affirmative, and the minor term S, its *predicate,* undistributed. Therefore S must be undistributed in the conclusion, of which it is subject; that is, the conclusion must be particular.

Exercise III

1. Prove from the general rules of the syllogism the following special rules of the fourth Figure: (*a*) If the major premiss is affirmative, the minor must be universal; (*b*) If the minor is affirmative, the conclusion must be particular; (*c*) If either premiss is negative, the major must be universal.

2. Show (from the general rules) what can be inferred as to the Figure and Mood of a valid syllogism—

   (*a*) When we know only that its conclusion is a universal affirmative.

   (*b*) When we know only that its major premiss is a particular affirmative.

   (*c*) When we know only that its minor premiss is a particular negative.

   (*d*) When we know only that its conclusion is a universal negative.

3 Given that the middle term of a valid syllogism is twice distributed, show (from the general rules) what can be inferred as to its conclusion.

4. Given that the major term of a valid syllogism is predicate in the major premiss, show (from the general rules) what can be inferred as to the minor premiss.

5. Given that the minor term of a valid syllogism is predicate in the minor premiss, show (from the general rules) that the conclusion cannot be a universal affirmative.

6. Why cannot the proposition **O** stand as a premiss in the first, as a major in the second, as a minor in the third, or as a premiss in the fourth Figure?

6. Reduction of Syllogisms

We have already quoted the technical names which have been given to the valid Moods in each of the four Figures. Each name of a Mood contains the vowel-symbols of its constituent propositions in their correct order. The following mnemonic, which has come down to us (with slight changes) from the medieval logicians, was devised,

as an aid to remembering the names, by fitting them into Latin hexameters:—

> Barbara, Celarent, Darii, Ferioque, prioris;
> Cesare, Camestres, Festino, Baroco, secundae;
> Tertia Darapti, Disamis, Datisi, Felapton,
> Bocardo, Ferison, habet; quarta insuper addit
> Bramantip, Camenes, Dimaris, Fesapo, Fresison.

These names, however, were not invented merely to record the symbols of the constituent propositions in all the valid Moods. We must explain why they were invented.

They were invented on the basis of a long-established tradition which goes back to Aristotle. Aristotle, for reasons which we cannot here discuss, believed that the essential nature of all true reasoning can be expressed, and is best expressed, in the form of the first Figure of the Syllogism in which a general rule and a related fact coming under it are brought together and the conclusion seen in the combination. This form of reasoning he held to be clear and conclusive above all other forms. Hence he called the first the "perfect Figure," and the others, since their conclusive character was in his view less evident, he called the "imperfect Figures." For this reason he devised a logical process which has come to be called, in English, **Reduction** (*i.e.* reduction to the first Figure). Its rule is: transform the premisses of the "imperfect" syllogism in such a way that the same conclusion may be drawn from them in one of the valid Moods of the first Figure. The transformation is affected (*a*) by Conversion of one or both of the premisses; (*b*) by transposing them, if necessary, in order to keep to the correct form of the first Figure.

Now the names of the various Moods in the "imperfect" Figures, as we have quoted them, and as they stand in the mnemonic lines, are not only the means of indicating,

by their three *vowels*, the quantity and quality of the major premiss, the minor, and the conclusion. Some of the intermediate consonants indicate the processes by which reduction is affected. The significant consonants are *s*, *p*, *m*, and *c*, and also the initial letters of the names, B, C, D, F. (1) The letter *s*, except when it is the last letter of the name (*i.e.* in *Camestres*, *Disamis*, *Camenes*), indicates that the proposition denoted by the preceding vowel is to be converted *simply*. (2) The letter *p*, except when it is the last letter of the name (which occurs only in *Bramantip*), indicates that the universal affirmative proposition denoted by the preceding vowel is to be converted *per accidens* (*i.e.* to a particular affirmative). (3) The letter *m* indicates that the premisses of the " imperfect " syllogism are to be transposed, the major becoming the new minor, and *vice versa*. (4) If there is an *s* or *p* at the end of the name of the " imperfect " syllogism, it means that the new syllogism in Figure I does not give a conclusion *identical* with that of the " imperfect " syllogism, but one from which the latter may be derived by conversion, simple or *per accidens*. (*e*) The initial letter, B, C, D, or F, of the name of the " imperfect " syllogism, shows the Mood in Figure I in which the new premisses give a valid conclusion.

For example: to " reduce " *Camestres*. This is in Figure II:—

All P is M,
No S is M,
∴ No S is P.

The first *s* in the name means that the original minor premiss is to be converted simply; the *m* means that the original premisses are to be transposed. The *C* indicates that from the new premisses, thus obtained, we are to draw the conclusion in *Celarent*, Figure I; and the second

s means that if we convert this conclusion simply, we shall get our original conclusion. Therefore, convert the original minor (*i.e.* in *Camestres*) and transpose the premisses:—

No M is S,
All P is M,

from which in *Celarent* the conclusion is—

No P is S,

from which by simple conversion—

No S is P,

which is the original conclusion.

The operation of which we have just given a typical example (direct application of conversion and transposition), has been called "direct reduction." Now Aristotle did not recognise any process of immediate inference except conversion; and under this limitation it is not possible to reduce *Baroco* (Figure II) and *Bocardo* (Figure III) directly. But there is no logical reason for abiding by this limitation.[1] *Baroco* can be "reduced" to *Ferio* by contrapositing its major premiss and obverting its minor:—

All P is M,
Some S is not M;
∴ Some S is not P.

Contraposit the major (*i.e.* first obvert, then convert), and obvert the minor:—

No non-M is P,
Some S is non-M,

[1] For this reason, two new mnemonic names have been invented: *Faksoko* for *Baroco* and *Doksamosk* for *Bocardo;* in these, *s* means "convert" and *k* means "obvert"; so that *ks* means "contraposit."

from which the conclusion in *Ferio* is—

Some S is not P.

And *Bocardo* can be reduced to *Darii* by contrapositing its major, transposing the premisses, and taking the obverted converse of the new conclusion. Take a syllogism in *Bocardo*:—

Some M is not P,
All M is S;
∴ Some S is not P.

Applying immediate inferences and transposition as indicated above, we get the new premisses—

All M is S,
Some non-P is M,

from which the conclusion in *Darii* is,

Some non-P is S,

from which, by conversion and obversion, we get:—

Some S is not P.

On the Aristotelian view, however, these two Moods have to be "reduced" to Figure I by a different process, called "indirect reduction," and indicated by the letter *c* in their names. Assume the falsity of the conclusion (of *Baroco*, or *Bocardo*, as the case may be), *i.e.* the truth of its contradictory; take this contradictory with one of the original premisses, and make them premisses of a new syllogism in *Barbara* (which, being in the first Figure, is known to be valid): the conclusion of the new syllogism will be incompatible with the other premiss of the original syllogism. Hence either the original conclusion is true or one of the original premisses is false; and, since on the

Aristotelian view the premisses are always assumed to be true, we can only accept the former alternative:—

(*a*) Reduce *Baroco* indirectly:—

$$\begin{cases}\text{All P is M,}\\ \text{Some S is not M.}\end{cases}$$
$$\therefore \text{Some S is not P.}$$

If this conclusion is false, its contradictory must be true; that is:— All S is P.

Make this the minor of a new syllogism with the original major:—

$$\begin{cases}\text{All P is M,}\\ \text{All S is P,}\end{cases}$$

from which the conclusion in *Barbara* is,

All S is M,

which contradicts the original minor. Therefore All S is M is false, and if so one of its premisses must be false. This can only be the assumed premiss All S is P; and if this is false, Some S is not P, the original conclusion, is true.

(*b*) Reduce *Bocardo* indirectly:—

$$\begin{cases}\text{Some M is not P.}\\ \text{All M is S;}\end{cases}$$
$$\therefore \text{Some S is not P.}$$

Take the contradictory of this conclusion with the original minor and draw a conclusion from them in *Barbara*:—

$$\begin{cases}\text{All S is P,}\\ \text{All M is S.}\end{cases}$$
$$\therefore \text{All M is P.}$$

This new conclusion must be false, for it contradicts the original major; hence its assumed premiss All S is P is false—*i.e.* the original conclusion, Some S is not P, is true.

It must be distinctly understood that Aristotle's theory of the superiority of the first Figure is fundamentally mistaken; and, so far as the process of "reduction" rests on a theory of the inferiority of the second and third Figures, the whole process is superfluous and even misleading. We have seen that the three Figures rest upon three different and independent principles of reasoning; and each Figure has its own special rules for the validity of its own valid Moods. We have explained the Aristotelian theory and practice of "reduction" because of its historical importance. And we shall retain the term "reduction," but we shall use it without any special reference to the first Figure, and merely as a technical name for the process of transforming (when possible) a syllogism valid in one Figure to an equivalent syllogism valid in some other Figure, by the application of immediate inference (eduction). As an exercise in mechanical manipulation, or rather, in "semi-mathematical" manipulation, the process is not without its utility. In some cases it is comparatively easy; thus, *Ferio* may be transformed into *Festino* by simply converting its major, into *Ferison* by simply converting its minor, and into *Fresison* by simply converting both its major and its minor. In other cases, obversion or contraposition may be required; thus *Darii* may be transformed into *Ferison* by obverting its major, simply converting its minor, and taking the obverse of the new conclusion (the symbol P′ means non-P):—

Darii.	*Ferison.*
M *a* P,	M *e* P′
S *i* M;	M *i* S;
∴ S *i* P.	∴ S *o* P′;
	∴ S *i* P.

But such transformations must not be regarded as throwing any light on the real nature of the reasoning in the respective Figures.

A further observation is necessary on the Aristotelian view of the first Figure. Aristotle formulated a self-evident principle or axiom which he rightly believed to apply *directly* to the first Figure only, and which later logicians, long after his time, called the *dictum de omni et de nullo*. They stated it thus: "Whatever is predicated, affirmatively or negatively, of the whole of a class, must be predicated, affirmatively or negatively, of everything contained under that class." The affirmative predication about the class is *de omni*, the negative *de nullo*; and the application of this to the rules of the first Figure, on the class-interpretation of the constituent propositions, is obvious. The major premiss makes a statement about all of a class, so that it must be universal, and may be negative; the minor asserts that a given case comes under that class, so that it must be affirmative; and the conclusion makes the original statement of the given case.

Aristotle, owing to his theory of the superiority of the first Figure, made no attempt to formulate self-evident principles or axioms for the second and third Figures; but such axioms can be stated, and have been stated by modern logicians. The axiom on which the reasonings in the second Figure depend has been expressed thus: "If a certain attribute can be predicated (affirmatively or negatively) of every member of a class, then any subject of which it cannot be so predicated, does not belong to that class"; or thus: "If every member of a class *has* (or, has not) a certain attribute, then any individual or individuals which *have not* (or, have) that attribute must be excluded from that class." This may be called the *dictum de diverso*. The axiom in which reasonings in the third Figure depend, called the *dictum de exemplo*, and may be thus expressed: "If a certain attribute is predicated *affirmatively or negatively* of certain individuals, and the same individuals belong to a certain class, then the attribute can be predicated *negatively or affirmatively* of *only some* members of that class"; or thus: If certain individuals (M) *have* (or, have not) a certain attribute (P), and the

same individuals are included in a certain class (S), then only some members of that class *have not* (or, have) the attribute (P).

On the Aristotelian *dictum*, see Mellone, *Introductory Textbook of Logic*, Chapter VI., § 6, especially pages 181-2 (in this chapter the treatment of Figure I is based on Aristotle); and Joseph, *Introduction to Logic*, second edition, Chapter XIV. (a thoroughly reasoned statement). For a very valuable and independent critical exposition of the traditional view of the syllogism, see W. E. Johnson, *Logic*, Volume II., Chapter 4.

7. Abridged Syllogisms

Syllogistic reasonings abound in ordinary discourse, but rarely, if ever, outside books on Logic, are such reasonings fully and formally expressed (see our observations on the first Figure, page 114). The natural tendency of speech is to state explicitly no more than is actually required for conveying to another mind what is meant. Hence we frequently find a genuinely syllogistic argument expressed verbally with the omission of one or other of the premisses or, more rarely, with the omission of the conclusion.

Some logicians have proposed to use the term **enthymeme** in a strict technical and formal sense, to signify a formally valid syllogism in which one premiss, or the conclusion, is not expressed. For example, the syllogism which when fully expressed is stated thus: "All religious wars are fought out with the greatest pertinacity and bitterness; the Thirty Years' War was a religious war: hence its length and bitterness"—may be expressed as an enthymeme in three ways: (*a*) The Thirty Years' War was long and bitter; for it was a religious war" (major premiss omitted; enthymeme of the "first order"); (*b*) "The Thirty Years' War was long and bitter, for all religious wars are so" (minor premiss omitted; enthymeme of the "second order"). (*c*) "All religious wars are long and bitter, and the Thirty Years' War was a religious war" (conclusion omitted; enthymeme of the "third order").

We shall use the term "enthymeme" in a wider sense to mean any kind of abridged or condensed syllogism whether valid or not. Jevons pointed out that even a single proposition may have a syllogistic force if it clearly suggests a second premiss which thus enables a conclusion to be drawn. The saying that "Men who have no rights cannot justly complain of any wrongs" seems to be a case in point. There are few people who have not felt wronged at some time or other, and they would therefore be likely to argue, whether upon true or false premisses, as follows:—

> *Men who have no rights cannot justly complain of any wrongs;*
> *We can justly complain;*
> *Therefore we are not men who have no rights* (i.e. *we have rights*).

In ordinary reasonings, again, we often find that a general principle is vaguely hinted at, or a subject is referred to a class the attributes of which are supposed to be definitely known. Thus:—

> *He was too ambitious to be scrupulous in his choice of means.*
> *He was too impulsive not to have made many blunders.*

Each of these sentences contains a conclusion and an "enthymematic" argument in support of it. We are supposed to have in mind a definite idea of the degree of ambition at which a man ceases to be scrupulous, or the degree of impulsiveness which is incompatible with accuracy.

Even a *question* may be a rhetorical device suggesting an argument: "Why be ashamed of a mistake? All men

are fallible." The *question* is equivalent to the statement "no mistakes are things to be ashamed of"; this is the conclusion. The given premiss "All men are fallible" must be re-stated so as to connect it with the conclusion: "A mistake is what all men are liable to." This contains the subject of the conclusion, and is therefore the minor premiss; it is universal, for it means to refer to every instance of "a mistake." The syllogism thus becomes:—

What all men are liable to is not a thing to be ashamed of;
A mistake is what all men are liable to;
Therefore no mistakes are things to be ashamed of.

This is valid in *Celarent*, Figure I, provided we admit that the words "liable to" *mean the same thing in the two premisses;* otherwise, we have ambiguous middle.

We shall now give some illustrative examples of the expression of arguments in strict syllogistic form. We shall begin with comparatively simple examples.

(1) *Name the Figure and Mood of each of the following syllogisms:* (a) "*Some M is S, no P is M, therefore some S is not P*"; (b) "*All S is M, no M is P, therefore no S is P.*"

This question is simply a reminder that a syllogism is not invalid merely because the major premiss stands in the second place instead of the first (as usual). In the given syllogisms, (*a*), which looks like a Mood of Figure I, is *Fresison* in Figure IV; and (*b*), which looks like Figure IV, is *Celarent* in Figure I.

(2) "*Every book is liable to error, and every book is a human production; therefore all human productions are liable to error.*"

The conclusion, though true, does not follow from the given premiss. It is an attempt to get a universal conclusion in *Darapti*, Figure III. Invalid by illicit minor.

(3) "*No true poets are persons destitute of imagination; some persons destitute of imagination are good logicians; therefore some true poets are not good logicians.*"

The premisses are those of *Fresison*, Figure IV, but the conclusion is invalid by illicit major. The correct conclusion is, " some good logicians are not true poets "; and the given conclusion cannot be derived from this by conversion.

(4) "*Every candid man acknowledges merit in a rival; every learned man does not do so; therefore every learned man is not candid.*"

Here the statement " every learned man does not do so " means logically " some learned men are not persons who acknowledge merit in a rival (**O**); and the suggested conclusion means " some learned men are not candid " (**O**). Thus the syllogism is:—

All candid men are persons who acknowledge merit in a rival,
Some learned men are not persons who acknowledge merit in a rival;
Therefore some learned men are not candid.

Valid in *Baroco*, Figure II.

(5) "*Any philosopher may be a great lawyer and statesman; for Bacon was a great lawyer and statesman, and was also a philosopher.*"

This argument attempts to base a conclusion on an instance or example (that of Bacon); hence it is of the nature of Figure III; the example (in this case an

individual) constitutes the middle term; and the premisses are:—

> *Bacon was a great lawyer and statesman,*
> *Bacon was a philosopher;*

from which the correct conclusion is in *Darapti*, Figure III:—

> *Some philosophers are great lawyers and statesmen,*

which means that there is no incompatibility between "being a philosopher" and "being a great lawyer and statesman." But what is the logical meaning of the given conclusion about "*any* philosopher"? "Any," thus used without qualification, means "all"; and the given conclusion is an attempt to get a universal result from *Darapti* and is invalid by illicit minor.

(6) "*A statesman may be a great writer; for such are Beaconsfield, Gladstone, Morley, and others.*"

Here, again, an example is appealed to in support of a conclusion, hence the principle is that of Figure III. In this case, the "example" consists of a *group of instances*, "Beaconsfield, Gladstone, Morley, and others." Logically, this group of instances constitutes a *Singular collective term*, and, as a Singular term, is technically universal; the syllogism is:—

> *Beaconsfield, Gladstone, Morley, and others are persons who were great writers,*
> *They are persons who were statesmen;*
> *Therefore some who were statesmen are persons who were great writers.*

Valid in *Darapti*, Figure III.

(7) "*Only members of the Society took part in the discussion. You must have done so, for you are a member.*"

We know that the proposition " Only S is P " does *not* warrant the statement that "*All* S is P"; it means logically " No non-S is P " *or* " All P is S." The latter form is more convenient; and the given premisses are:—

All who took part in the discussion are members of the Society;
You are a member of the Society.

Undistributed middle; no conclusion possible (attempt to get a conclusion from two affirmative premisses in the form of Figure II; see page 128).

(8) "*This document cannot be genuine, or it would have been referred to by the supposed author's contemporaries.*"

We take " genuine " to mean " really written by the man to whom it is ascribed or whose name (as author) it bears." In this example, the major premiss is omitted. It is meant to be " All genuine documents are referred to by the supposed author's contemporaries "—a doubtful and debatable statement. But if we accept it, the argument is valid in *Camestres*, Figure II, the minor and conclusion being respectively " This document is not referred to by the supposed author's contemporaries," and " This document is not genuine."

(9) "*Those who achieve great ends are happy; hence young people cannot be happy, for great ends cannot be realised in youth.*"

This is an attempt to make a distinction by means of Figure I instead of Figure II:—

All who achieve great ends are happy,
No young people achieve great ends;
Therefore no young people are happy.

Invalid by illicit major.

(10) "*No belief is worth dying for, since none is certain.*"

The given premiss "none is certain" may be expressed in the form "All beliefs are uncertain." The conclusion "no belief is worth dying for" shows us that the major term is "worth dying for," and that the omitted premiss is the major premiss:—

Nothing uncertain is worth dying for,
All beliefs are uncertain;
Therefore no belief is worth dying for.

Valid in *Celarent*, Figure I.

(11) "*Few men know their own business, for it cannot be understood without a knowledge of things outside it.*"

Here the given minor premiss "it cannot be understood, etc.," shows what the assumed major premiss is: but we must alter the impersonal form of expression:—

None who are ignorant of things outside their own business are persons who know their own business;
Some men are ignorant of things outside their own business;
Some men are persons who do not know their own business.

Valid in *Ferio*, Figure I.

The modern doctrine of the enthymeme in Logic is in effect a caution, warning us to remember the ways in which in ordinary reasoning the real grounds of the argument are concealed. This is why fallacies are so often *hidden;* an argument is based upon some unexpressed assumption which will not bear examination.

We must now observe that syllogisms may be combined in various ways into "chains of reasoning." It does not often happen that a question can be settled by the aid of one syllogism. Its premisses may themselves require arguments in their support; this leads to a chain of syllogisms;

and as there is a natural tendency in ordinary discourse to abridge them, what we usually find is a chain of abridged syllogisms; we move on from fact to fact, or rather from statement to statement, without stopping to express all the stops definitely and explicitly in words. What is really important is to be able to recognise the logical character of such "chains," to analyse them into their constituent single syllogisms, and to discover their conformity or otherwise to the general rules of the syllogism.

The form of argument in which we pass steadily from one syllogism to another, making each conclusion as soon as it is established the premiss of a new syllogism, attracted the attention of logicians from the time of Aristotle onwards. They analysed only one kind of it, which they called a "Sorites." A "Sorites" is a chain of syllogisms in which all the propositions are expressed in strict logical form, and all the conclusions except the last are omitted. In the "Aristotelian" Sorites, the general form is: A is B, B is C, C is D; ∴ A is D (there may of course be three or more syllogisms implied). It progresses from terms of narrower to those of wider denotation; and (in addition to the conclusions) the minor premiss of every syllogism except the first is not expressed At the end of the sixteenth century it was pointed out by a German logician, Goclenius, that it is possible to proceed in the opposite order. The general form of the "Goclenian" Sorites is therefore. C is D, B is C, A is B; ∴ A is D. It "regresses" from terms of wider to those of narrower denotation, *i.e.* D, C, B, A; and (in addition to the conclusions) the major premiss of every syllogism except the first is omitted.

For the sake of clearness we add an analysis of the two forms.

Aristotelian Sorites.

A is B,
B is C,
C is D; ∴ A is D.

Analysis.

(1) $\begin{cases}\text{A is B (minor).}\\ \text{B is C (major).}\end{cases}$
A is C (conclusion).

(2) $\begin{cases}\text{A is C (minor).}\\ \text{C is D (major)}\end{cases}$
A is D (conclusion).

Goclenian Sorites.

C is D,
B is C,
A is B; ∴ A is D.

Analysis.

(1) $\begin{cases}\text{C is D (major).}\\ \text{B is C (minor).}\end{cases}$
B is D (conclusion).

(2) $\begin{cases}\text{B is D (major).}\\ \text{A is B (minor).}\end{cases}$
A is D (conclusion).

In these examples all the propositions are universal and affirmative. Particular or negative propositions are possible only at the beginning or the end In the "Aristotelian" form, only the first premise can be particular, and only the last premiss can be negative, for if any of the intermediate premisses were either negative or particular, the chain of connection would be broken The "Goclenian" Sorites proceeds in the reverse order; here only the first premiss can be negative, and only the last premiss can be particular.

To turn ordinary or colloquial reasonings into syllogistic form, and so to test their validity, is a valuable exercise in accuracy of thought. The reader should bear in mind the nature of the reasoning expressed in technical form in each of the first three Figures. If the given argument attempts to prove or disprove some attribute of a thing by applying a general rule or principle or bringing it under a higher class, the Figure is the *first*. If the argument attempts to separate and distinguish two things by reasoning from the fact that an attribute which is characteristic of one is absent in the other, the Figure is the *second*. If the argument attempts to establish a general or partly general statement by an instance or example (or by a group of examples), or if it attempts to deny such statement by means of a negative example, then the Figure is the *third*. Breaches of the syllogistic rules often occur through attempting to get a conclusion in the wrong Figure: arguing, for instance, in Figure I instead of Figure II or Figure III, or attempting to argue in Figure II from resemblances (see page 128).

In the application of a general principle to a particular case, a fallacy is possible which is far from uncommon. It consists in assuming that what holds good as a general rule will hold good under some special circumstances which may entirely alter the case. For example, when it is admitted that culture is good, a disputant may go on to argue as if the admission applied to some particular kind

of culture—scientific, or classical, etc.; or when it agreed that every man has a right to inculcate his own opinions, it is argued that a magistrate is justified in using his power to enforce his own political views. But we cannot infer of his special powers as a magistrate what is true only of his general rights as a man. Ancient logicians called this the fallacy of arguing *a dicto simpliciter ad dictum secundum quid*: the general principle being accepted *simpliciter*, without qualification, and then applied *secundum quid*, to some particular case in special circumstances which do not really come under the rule. There is a converse form of this fallacy—that of arguing *a dicto secundum quid ad dictum simpliciter*—getting assent to a statement with a qualification (*secundum quid*) and proceeding to argue as if it had been conceded without qualification (*simpliciter*); as when it is shown that the syllogism is useless for a certain stated purpose, and the resulting conclusion is used as if it proved that the syllogism is useless for any purpose. When expressed in syllogistic form these fallacies produce syllogisms with four terms in Figure I or in Figure III respectively.

Exercise IV

State each of the following arguments in syllogistic form, and examine its validity.

1. If he did not steal the goods, why did he hide them, as no thief fails to do?

2. This book must have been read, because the pages are cut.

3. Some of the books have evidently not been used, for the pages have not even been cut.

4. A little knowledge is a dangerous thing, hence I had better not try to learn Logic.

5. A little knowledge is not a dangerous thing unless we imagine it to be greater than it really is.

6. I do not derive my opinions from books, for I have none.

7. He must be a Buddhist, for all Buddhists hold these opinions.

8. Wind and weather are unpredictable, although they are governed by laws.

9. The uncritical acceptance of the views of one's teachers is no real education; it fails to develop the power of independent judgment, which is the essence of real education.

10. Emperors may be good men, and good men may be Emperors, for Marcus Aurelius was both a good man and an Emperor.

11. We know that the policy was mistaken, for otherwise it would not have failed.

12. It is impossible to prove that persecution is justifiable if you cannot prove that some non-effective measures are justifiable; for no persecution has ever succeeded.

13. All Law is an abridgment of liberty and consequently of happiness.

14. Anger is not always an evil, for moral indignation involves anger and is sometimes a real good

15. Every truth is worthy of being known, but not every truth is directly useful.

16. The right to say what one believes to be true is worth dying for, because all human progress depends upon it.

17. Idiots cannot be men, for man is a rational being.

18. Apart from feelings due to the internal organs of the body, emotion can have no existence. Therefore every emotion is wholly constituted by such feelings.

19. I speak not from mere theory; there exist at this moment practical illustrations of what I say.

20. All logicians know how to reason well, but some logicians do not reason well; hence some who know how to reason well do not do it.

For a selection of longer passages involving reasoning mainly deductive in character, see Welton and Monahan, *Intermediate Logic*, third edition, revised by E. M. Whetnall, pages 477-9, and F. C. Bartlett, *Exercises in Logic*, Ch. XII. (pages 127 ff.). The following observations on "illustrations of the syllogism" are of interest. "There are two opposite tendencies in the choice of

illustrations of the syllogism, both of which, in my view, should be avoided. The first is to select examples composed of propositions each of which is universally accepted as true But such illustrations hinder the learner from examining the validity of the inferential process from premisses to conclusion, since he is apt to assume validity because of his familiarity with the propositions as generally accepted. The opposite course, which we find amusingly illustrated by Lewis Carroll, is to select propositions which are obviously false. But this leads the learner to regard the syllogism merely as a kind of game, and as having no real significance in actual thought-procedure. It is preferable therefore to select propositions which are dubious, or which are affirmed by some persons and denied by others. Of such propositions, important kinds are (1) those which deal with political, ethical, or similar topics in *general*, *e.g.* 'Lying is sometimes right,' 'All countries that adopt free-trade are prosperous,' 'The suffrage should not be extended to uneducated persons'; (2) those which exercise the faculty of judgment (appreciation) . . . upon some *individual* case, *e.g.* 'This man is untrustworthy,' 'Esau is a more lovable person than Jacob.'" (W. E. Johnson, *Logic*, Vol. II., pages 77-8.)

8. Hypothetical Syllogisms

We have now to explain the possible forms of syllogisms in which one or both of the premisses is a hypothetical proposition. The reasoning in such syllogisms is purely deductive.

We have already discussed the forms which hypothetical propositions may assume (see Chapter IV., page 72). The traditional terminology is in general use in elementary Logic, and we shall keep to it; though the terminology employed by recent logicians has been explained for the reader's information (see Chapter IV., page 83). The forms of the hypothetical proposition which most frequently occur are these: "If S is P, Q is R," and "If S is P it is Q." We pointed out (page 72) that logically these two forms are the same. Take the latter. This asserts that if "S is P" is true, then "S is Q" is true. In other words, the compound proposition asserts that the simple proposition "S is P" *implies* the simple proposition

" S is Q." What is asserted, is this relation of *implication*. The assertion may be *based upon* scientific investigation, or upon mathematical calculation, or upon the respective connotations of two abstract terms, or otherwise; but the logical relation asserted is one of " implication."

The hypothetical proposition has been called " conditional " because (to take again the form used above) Q is predicated of S under a condition : " S is Q " is true under the condition that " S is P " is true. This is certainly the case, and as a matter of terminology this use of the word " condition " is admissible. In ordinary discourse the condition may be introduced not only by " if," but by an equivalent phrase—*e.g.* " suppose that," " granted or provided that," " wherever," " whenever." The part of the hypothetical proposition which expresses the condition or supposition is called the **antecedent**; the other, implied by the antecedent, is called the **consequent.** Hence a convenient abbreviated symbolic form for the hypothetical proposition is " If A, then C "; and corresponding to this, are the forms " If not A, then C," " If A, then not C," and " If not A, then not C."

We must observe that the hypothetical proposition is logically universal: " Wherever and whenever A is true, C is true." There is no condition or qualification attached to the *implication* which is asserted. " If there is a real connection between the antecedent and the consequent, the proposition is universal; and if there is no such real connection, the assertion should be expressed in the categorical form, not in the hypothetical form. It will be found that in a " particular " hypothetical proposition the particle " if " really means " when " (not " whenever "). The difference between *if* and *when* is this: *if* introduces the *condition* of a certain result; and a real condition is universal, and is best expressed in a hypothetical

proposition; *when* introduces *instances* of a certain result, and is best expressed in the categorical form of proposition."[1]

Thus, "If A, then *sometimes* C" is a "particular hypothetical proposition," and its logical meaning is best expressed in the form "Some cases of A are cases of C," or more briefly, "Some A's are C." When we wish to express the abstract relation of implication, the hypothetical form is logically the correct one: "If A, then C"; "If a triangle is equilateral, it is equiangular"; "If the temperature of a gas is raised, its volume is increased" (the pressure remaining unchanged); "If the supply of a commodity falls short of the demand, the price tends to rise" (in the absence of Government control). It is also true that any universal hypothetical proposition may be transformed into a categorical proposition, but this alters its logical character. Instead of thinking of the connection between A and C, we think of particular instances of A and C: "All A's are C's"; "Every case of A is a case of C"; "All equilateral triangles are equiangular"; "Any gas, the temperature of which is raised, increases in volume"; "Any commodity, the supply of which falls short of the demand, tends to rise in price."

Hypothetical syllogisms are of two kinds: usually distinguished as "pure," in which both premisses are hypothetical, and "mixed," in which the major premiss is hypothetical and the minor categorical.

(1) The following examples illustrate the nature of "pure" hypothetical syllogisms:—[2]

If the rays of light coming from the fixed stars are subject to gravitation they will be bent by planets near their path to the earth;

[1] Wolf, *Textbook of Logic*, Ch. XII., § 2, p. 125.
[2] See Wolf, *Textbook of Logic*, Ch. XII., pp. 126-8.

If the rays of light, etc., are material they are subject to gravitation;
Therefore if the rays of light, etc., are material they will be bent by planets near their path to the earth.

Expressed in symbols, this syllogism is of the form:—

If B then C,
If A then B;
∴ If A then C.

Transformed into a categorical syllogism, it becomes *Barbara*, Figure I:—

All B is C,
All A is B;
∴ All A is C.

The following example has a negative major premiss:—

If a triangle is equiangular it is not right-angled.
If a triangle is equilateral it is equiangular;
Therefore if a triangle is equilateral it is not right-angled.

Transformed into a categorical syllogism it becomes *Celarent*, Figure I:—

No B is C,
All A is B;
∴ No A is C.

Fallacies may occur in "pure" hypothetical syllogisms; for example:—

If a man is guilty he is uncomfortable under cross-examination;
If a man is nervous he is uncomfortable under cross-examination;
Therefore if a man is nervous, he is guilty.

Expressed in symbols, this syllogism is of the form:—

If C, then B,
If A, then B;
∴ If A, then C.

Transformed into a categorical syllogism, it becomes:—

All C is B,
All A is B;
∴ All A is C,

which is invalid by undistributed middle.

(2) "Mixed" hypothetical syllogisms are more important because (as we shall see) they provide a formal expression for the relation of a scientific hypothesis to actually observed facts.

The minor premiss of a "mixed" hypothetical syllogism may affirm or deny the antecedent or consequent of the major; hence there are four numerically possible forms (where "A" in the minor means "affirmation of A," and "not A," "denial of A"; and so with "C"):—

a) If A, then C;
A;
∴ C.

(*b*) If A, then C;
Not C;
∴ Not A.

c) If A, then C;
Not A;
∴ Not C.

(*d*) If A, then C;
C;
∴ A.

Now it will be seen, on consideration, that the conclusions in (*c*) and (*d*) are invalid, and that there can be no valid conclusion in these forms. If we deny the antecedent, as in (*c*), we cannot therefore deny the consequent; because the latter may be true for other reasons; and if we affirm

the consequent, as in (*d*), we cannot therefore affirm the antecedent because the consequent may result from other reasons. Hence the rule for "mixed" hypothetical syllogisms is this: *Either affirm the antecedent or deny the consequent.* In the former case, as in (*a*) above, we have what is called a "constructive" hypothetical syllogism; in the latter, as in (*b*), a "destructive" hypothetical syllogism.

Our results may be summarised as follows, in tabular form:—

Hypothetical Syllogisms

"Pure" (both premisses hypothetical)		"Mixed" (minor premiss categorical)	
one premiss particular (really categorical)	Both premisses universal	Constructive (affirmation of antecedent)	Destructive (denial of consequent)

We shall now give concrete examples of "mixed" hypothetical syllogisms. We must remember that if the antecedent is negative ("not A") we *deny* it by simply affirming "A," and we *affirm* it by simply affirming "not A." Similarly, if the consequent is negative ("not C"), we *deny* it by simply affirming "C," and we *affirm* it by simply affirming "not C."

(1) *If the soul is uncreated, it is indestructible,*
The soul is uncreated;
Therefore it is indestructible.

Whatever we think of the material truth of the minor premiss, the argument from the stated implication is valid by "affirmation of the antecedent."

(2) *If men have obligations towards their friends, they have them towards their enemies;*
Men have obligations towards their friends;
Therefore they have them towards their enemies.

This also is valid by affirmation of the antecedent.

(3) In the following example, we have the affirmation of a negative antecedent: "It was agreed that unless the weather turned fine, we were to postpone the match; so, as the weather has not turned fine, the match must be postponed." In logical form, this becomes:—

If the weather does not turn fine, the match is to be postponed;
The weather has not turned fine;
Therefore the match is to be postponed.

(4) The following is an example in which the consequent is denied, and the denial of the antecedent is correctly inferred:—

If life is possible on the planet Mars, the planet has warmth sufficient for constructive metabolism[1]*;*
But the planet has not warmth sufficient;
Therefore life is not possible on it.

(5) In the following we deny a negative consequent by affirming its contradictory, and then correctly infer the denial of a negative antecedent (*i.e.* the affirmation of its contradictory):—

If the earth did not rotate on its axis, the winds that blow from the poles to the equator would not be deflected westward;
But they are deflected westward;
Therefore the earth does rotate.

[1] The process, in an organism or a single cell, by which nutritive material is built up into living matter.

Here, "the earth" is a Singular term, and "the winds that blow from the pole to the equator" is a *collective* Singular term. In this connection, when we infer the denial of the antecedent, or the denial of the consequent, we must remember that a proposition is logically denied by its *contradictory* not by its *contrary* :—

If all men were capable of perfection, some would have attained it;
But none have attained it (contradictory of consequent);
Therefore none are capable of it (contrary of antecedent).

Here the correct conclusion is, "Some are not capable of it" (the *contradictory* of the antecedent).

Now consider the following examples:—

(6) *If the study of Logic furnished the mind with a multitude of useful facts, like other sciences, it would deserve to be cultivated;*
But it does not furnish the mind with a multitude of useful facts;
Therefore it does not deserve cultivation.

This conclusion does not follow from the premisses; for the acquiring of a multitude of useful facts is not the only ground on which the study of a science can be recommended. To correct and exercise the powers of judgment and reasoning may be regarded, for example, as a sufficient justification of logical study. The fallacy is that of denying the antecedent.

(7) *If a man's character is avaricious, he will refuse to give money for useful purposes;*
This man refuses money for such purposes;
Therefore this man's character is avaricious.

But we are not entitled to infer this from the premisses; for there may be many good reasons why he refuses, although his character is not avaricious. The fallacy is that of affirming the consequent.

The real structure of the "mixed" hypothetical syllogism is now evident. The major premiss affirms only that the logical relation of implication holds between an antecedent proposition A and a consequent proposition C. It says nothing about *instances* of the truth of A or of C. We may know that A implies C, *i.e.* "If A, then C," without knowing that "A is true, therefore C is true." To say that "if the barometer falls, the weather will be bad" is not the same thing as to say "the barometer is falling and so the weather will be bad." But when, independently of the major, we know the truth of the minor, "A is true," "the barometer is falling," then we may assert the conclusion. Similarly, from the minor premiss alone, "A," or "not C," we cannot derive the conclusion, "C," or "not A," unless the relation of implication is admitted to hold between them, *i.e.* unless the major premiss is conceded as well as the minor.

9. The Disjunctive Syllogism

The disjunctive syllogism has a disjunctive (or alternative) major premiss and a categorical minor and conclusion.

We have already seen that the traditional treatment of the disjunctive proposition fails to distinguish between the cases in which the alternatives are mutually exclusive and together exhaustive of the possibilities, and the cases in which they are not so. Propositions of the former kind are called by modern logicians "disjunctive" propositions in the strict sense of the word; and those of the latter kind are called "alternative" propositions. The symbolic

forms are: " Either S is P or Q is R "; " S is either P or Q "; or, most briefly, and using different symbols, " Either A or B " (*i.e.* either A is true, or B is true). It is, of course, impossible to tell from the symbolic form of the proposition whether it is strictly disjunctive or merely alternative; this can only be ascertained by consideration of propositions actually asserted.

In ordinary thinking, speaking, and writing, we frequently do not mean the alternatives to be mutually exclusive, or, perhaps even more frequently, we do not consider whether they are exclusive or not. Take the following instances: " All the men in this college either boat or play cricket "; " A good book is valued either for the usefulness of its contents or the excellence of its style"; " Either the witness is perjured, or the prisoner is guilty." In all these propositions, the meaning is merely that if one alternative *does not* hold, then the other does hold. In such cases we are not concerned to deny that both the alternatives may be true.

The major premiss of a disjunctive syllogism, in such cases, is an alternative proposition, " Either A or B," and there are four possible minors, " A," " not A," " B," " not B," where " A " and " B " are universal affirmative propositions, and " not A " and " not B " universal negatives. Hence four forms are numerically possible (taking first those in which the minor is negative):—

(*a*) Either A or B;	(*b*) Either A or B;
Not A;	Not B;
∴ B.	∴ A.

Thus we may resolve the alternative proposition into two hypotheticals (taken together):—

$$\begin{cases} (a) \text{ If not A, then B;} \\ (b) \text{ If not B, then A.} \end{cases}$$

But there is no conclusion from either of the following pairs of premisses:—

(*c*) Either A or B;
A;

(*d*) Either A or B;
B;

because A and B may be true together; so that the affirmation of A does not enable us to infer anything as to the truth of B, and *vice versa*.

In order to assert a strictly disjunctive proposition, in which the alternatives are mutually exclusive and together exhaustive of the possibilities, we must have a considerable amount of knowledge about the subject. Even to say such a thing as this, "you must either pay a fine or go to prison," implies a knowledge of the legal bearings of the circumstances as a whole. "A line must be either straight or curved" implies mathematical knowledge of the meaning of straight and curved, and the relation between them. "This tree is either an oak or an ash" implies some knowledge of both these varieties, and a comparison of that knowledge with the given instance. In fact, a strictly disjunctive proposition implies knowledge of a genus and species codified according to the rules of logical Division (see Chapter III., page 49) within which the Subject of the disjunction falls; so that there may be more than two possibilities. Thus, when we know that S belongs to the genus P, and also that P is logically divided into the species p_1, p_2, p_3, we can assert that S is either p_1 or p_2 or p_3; if it is p_1 it is not p_2 or p_3, and if not p_1 or p_2 then it is p_3; and so on. The multiplication of the possibilities greatly increases the complexity of the reasoning but does not alter its logical character. To take a simpler case: when we know that a certain individual is a British subject, then we can say "he is either a British-born subject or a naturalised British subject, and if he is

the one, he *is not* the other." So that with a strictly disjunctive major premiss, we may have conclusions not only in the forms (*a*) and (*b*) given above, but also in (*c*) and (*d*):—

(*c*) Either A or B;	(*d*) Either A or B;
A;	B;
∴ Not B.	∴ Not A.

10. Dilemmas

A dilemma is a syllogism with one premiss disjunctive and the other hypothetical.

In practical life we are said to be in a dilemma when we have only two courses open to us, and both will have unpleasant consequences. So, in Logic, the dilemma shuts us up to a choice between two admissions.

The structure of the dilemma will be apparent from the following rules and examples:—

(1) The **major premiss** is a hypothetical proposition:—
- (*a*) with more than one antecedent;
- (*b*) or with more than one consequent;
- (*c*) or with more than one of both, so as to be two hypotheticals combined.

(2) The **minor premiss** is a disjunctive proposition.

(3) The **conclusion** is either a categorical or a disjunctive proposition, according as the hypothetical major has only one antecedent (or consequent) or more than one. The dilemma is said to be simple or complex according as its conclusion is categorical or disjunctive.

(4) The essentials of the dilemma are the **plurality of antecedents or of consequents** in the major, and the **disjunctive minor.**

Hence there are four possible forms of the dilemma:—

(*a*) *Simple Constructive.*

If A or if B, then C;
Either A or B;
∴ C.

(*b*) *Simple Destructive.*

If A, then both B and C;
Either not B or not C;
∴ Not A.

(*c*) *Complex Constructive.*

If A, then B; and if C, then D;
Either A or C;
∴ Either B or D.

(*d*) *Complex Destructive.*

If A, then B; and if C, then D;
Either not B or not D;
∴ Either not A or not C.

These various forms may be thus illustrated. The following effective examples of the two kinds of simple dilemma are given by Professor Wolf:—

(*a*) *If the miners have to work longer or to earn less, they will be dissatisfied; but they must accept either longer hours or reduced wages; therefore the miners will be dissatisfied.*

(*b*) *If the coal industry were in a sound condition the miners and the mine-owners would be contented; but either the miners or the mine-owners are discontented; therefore the coal industry is not in a sound condition.*

The complex constructive dilemma is a common form:—

(*c*) *If I cross the field I shall meet the bull, and if I go up the lane I shall meet the farmer; but either I must cross the field or I must go up the lane; therefore either I must meet the bull or I must meet the farmer.*

In this case the dilemma is an analysis of a practical situation. A famous example is the fallacious dilemma in which the custodians of the Alexandrian Library are said to have been put by Caliph Omar in 640 A.D. "If your books are in conformity with the Koran, they are superfluous; and if they are at variance with it, they are pernicious."

The complex destructive dilemma is less common:—

(*d*) *If he were clever he would see his mistake; and if he were candid he would acknowledge it; but either he does not see his mistake or he will not acknowledge it; therefore either he is not clever or he is not candid.*

If it is properly constructed, the dilemma is logically sound; but many fallacies have been put into this form. Fallacy may arise from a faulty major or a faulty minor premiss. In the major premiss the antecedent, or the consequent, may be false *in fact*, or the asserted connection between them may be false. In the minor premiss—where the fallacy usually lies—the antecedent of the major may be denied or the consequent affirmed; or the alternatives may not be exclusive or not exhaustive. This last is the most common source of hidden fallacy in the dilemma.

It is seldom possible to find instances where two alternatives exhaust all the possible cases, unless indeed one of them be the simple negative of the other in accordance with the law of excluded middle. Thus if we were to argue that "if a pupil is fond of learning, he needs no stimulus, and if he dislikes learning, no stimulus will be of any avail; but as he is either fond of learning or dislikes it, a stimulus is either needless or of no avail," we evidently assume improperly the disjunctive minor premiss. Fondness and dislike are not

the only two possible alternatives, for there may be some who are neither fond of learning or dislike it, and to these a stimulus of some kind may be desirable. Almost anything can be proved if we are allowed thus to pick out two of the possible alternatives which are in our favour, and argue from these alone.

The most famous illustration of these observations is the ancient fallacy known as *Ignava Ratio*, the "lazy argument": "If it be fated that you recover from your present disease, you will recover, whether you call in a doctor or not; again, if it be fated that you do not recover from your present disease, you will not recover, whether you call in a doctor or not: but one or other of these contradictories is fated, and therefore it can be of no service to call in a doctor." On this, Professor Wolf observes: "If there is no such thing as fatalistic destiny, then everything might depend on the medical aid called in, in case of illness. This is the common-sense view; but this possibility is entirely omitted from the argument—to say nothing of the possibility of a conditional destiny depending on certain measures being taken. The real possibilities may be indicated in the following scheme: A man's lot may be either (*a*) *destined*, or (*b*) *not destined;* and if *destined*, then the destiny may be either (1) *conditional* on certain steps being taken, or (2) *unconditional*. The above dilemma as a matter of fact is based on (*a*) (2) alone, and quietly ignores (*a*) (1) and (*b*) altogether."[1]

In the dilemma with respect to the Alexandrian Library, Caliph Omar tacitly assumed in the minor premiss that the doctrines of the Koran are not merely sound, but contain all that is really worth knowing. Or, to put it otherwise, he ignores the possibility that the books may contain

[1] Wolf, *Textbook of Logic*, Ch. XIV., p. 142.

useful matter on which the Koran does not touch. In other words, the alternatives given in the minor premiss are not exhaustive.

A faulty constructive dilemma may be "rebutted" by a dilemma which appears equally cogent, and appears to prove an opposite conclusion. As an example we may take a story which has come down to us of an Athenian mother who urged her son not to enter on public life, on the following grounds:—

> "*If you say what is just, men will hate you; and if you say what is unjust, the Gods will hate you. You must say one or the other; therefore you will be hated.*"

The son replied that he ought to enter on public life, giving the following reasons:—

> "*If I say what is just, the Gods will love me; and if I say what is unjust, men will love me; I must say one or the other; therefore I shall be loved.*"

In most of such cases, the so-called "rebuttal" is only apparent; the two dilemmas are equally fallacious. A complex constructive dilemma which is fallacious, may be "rebutted" thus: transpose the two consequents in the major premiss, turning each into its contradictory:—

(1) If A, then B; and if C, then D;
But either A or C;
∴ Either B or D.

(2) If A, then not D; and if C, then not B;
But either A or C;
∴ Either not D or not B.

An excellent brief account of hypothetical and disjunctive propositions and syllogisms will be found in Wolf's *Textbook of Logic*, Chh. XII., XIII., XIV.; see also Welton and Monahan, *Intermediate*

Logic, third edition, revised by E M. Whetnall, Ch. XVIII. (*cp.* Ch. VIII., pages 90-3). On the view of hypothetical and disjunctive reasoning advocated by F. H Bradley (*Principles of Logic*, second edition) and Bernard Bosanquet (*Logic*, second edition), see Mellone, *Introductory Textbook of Logic*, Ch. XI., and the references there given The view worked out and defended by Bradley and Bosanquet belongs to the philosophical Theory of Knowledge rather than to Logic. They take the essential forms of logical judgment as a clue to the structure of the real world.

11. Other Types of Deductive Reasoning

We have already explained the general nature of deductive reasoning (see page 104); and, so far, we have been discussing deduction as exemplified in the various kinds of syllogistic reasoning.

In the formal syllogism the copula of the propositions naturally expresses only the relation of subject and attribute; and though, as we have seen, it easily expresses the relation of genus and species (class inclusion and exclusion), it does not and cannot express all possible relations by the word "is." This ought to be evident even apart from what we pointed out when discussing the classification of propositions (Chapter IV., page 75). The ideas of space and time, of resemblance and difference, of quantitative magnitude, to mention only these, seem to involve distinct laws of thought, to create for themselves special forms of language, and to require special canons of Logic. In all these fields there is room for such relations of *implication* as demonstration requires; but the rules of class-reasoning, under the *dictum de omni et de nullo*, have no natural application; and the propositions which are involved differ entirely in their nature from those of the subject-attribute form. There are therefore many types of valid deductive reasoning which are non-syllogistic; and it is no longer customary to commit the absurdity of describing them as "irregular" arguments.

In this book it is only possible to give the simplest examples: but they are sufficient to illustrate the principles which are involved.

(1) **Relations of Identity.**—The most familiar examples of these are seen in propositions asserting quantitative equality: " A is equal to B, and B is equal to C, therefore A is equal to C "; or " A is equal to B, and C is equal to B, therefore A is equal to C." The reasoning involves a common factor; but this common factor is not a middle term in the strict syllogistic sense. Under the same head come reasonings based on *relations of affinity*: " Bernard is Helen's brother, and Muriel is Helen's sister; therefore Bernard is Muriel's brother "; or, again, " John is Mary's brother, Henry is Mary's son, therefore John is Henry's uncle." The common factor is not a syllogistic middle term. The *deductive system* (see page 105) is that of family relationships. We are not usually concerned with family relationships removed more than a very few degrees. Yet such relationships ramify into complex genealogical trees, and soon reach affinities for which we have no special names, because we do not need to refer to them in ordinary life. Their importance is seen when inheritance of title or property is at stake; and then the problem is to construct the system and draw from it the true conclusion as to hereditary right.

It is evident that all arguments based on relations of identity among particular individuals or places or things come under this head. A French Abbé of the Louis Quatorze period was conversing with some ladies in a *salon*; and he said: " Ah, ladies, a priest has strange experiences: my first penitent was a murderer." A few moments afterwards the principal nobleman of the neighbourhood entered the *salon*, and said, " Ah, here you are, Abbé; do

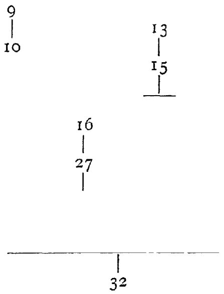
you know, ladies, I was the Abbé's first penitent, and, I warrant you, my confession astonished him ! "

Professor Wolf (*Textbook of Logic*, Ch VIII., and Ch IX., § 1) has classed these arguments from relations of identity, along with certain others, as " Mediate inference with a Singular middle term " This is suggestive, but the logical character of the relating factor in the propositions is as important as the logical character of the middle term; and we are now regarding such arguments from the point of view of the relating factors. We have said that they have no "middle term" *in the strict syllogistic sense*: this is shown by Professor Wolf (*op. cit.*, Ch. IX.), when he treats the ordinary syllogism as being characterised by a middle term which is logically *general.*

(2) **Relations of Space.**—In our experience, space appears as a form in which we always experience objects outside us. When we attend only to the *spatial relations* of the objects which we perceive or imagine, then we find that spatial relations form a deductive system within which inferences may be made. The inference takes the form of a construction, a diagram actually drawn or imagined, and the conclusion is either directly seen in the construction or derived from it by analysis. The following are simple examples of conclusions directly seen in the construction: " A is north of B, B is north of C; therefore A is north of C "; " Q is a certain distance south of R, and P is the same distance west of Q, therefore P is south-west of R." In such cases the conclusion appears immediately the construction is made. But in all geometrical reasoning, the principle is the same. When we prove that the angles at the base of an isosceles triangle are equal to each other, we may draw a diagram, mark its three angles by letters, and refer to it at every step of the demonstration; but the reasoning is concerned only with the purely spatial relations which the figure embodies, and which are seen in it; and not with the size, position, or any other accidental qualities of the figure as drawn. The conclusions of previous geometrical reasonings are appealed to at certain steps in the

reasoning which at bottom rests upon a number of "first principles"—definitions and axioms (self-evident propositions) about the nature of space.

Readers who are acquainted with elementary geometry may be interested in tracing the following logical "pedigree," in which the thirty-second proposition of the first Book of Euclid's *Elements* ("The three interior angles of every triangle are together equal to two right angles") is traced back to the first and fourth propositions[1]:—



[1] For this diagram I am indebted to F. W. Westaway's *Scientific Method,* fourth edition, page 165.

(3) **Relations of Time.**—Time, like space, appears in our experience as a form in which we always experience events outside us. The essential time-relations are simultaneity or co-existence, and sequence or before-and-after. All reasoning about the "chronological order" of events is evidently based on these relations. "Bacon lived before Locke and Locke lived before Hume, therefore Bacon lived before Hume." "If Wellington defeated Napoleon at the battle of Waterloo, he must have been contemporary with him."

Investigation of time-sequence may lead to conclusions in terms of cause and effect; but "cause and effect" is not mere temporal sequence: "War involves excessive expenditure; to meet this, heavy taxation must be imposed and large loans raised; to pay the interest on these loans the taxation has to remain heavy long after peace has been made; therefore, war leads to long-continued heavy taxation." But the fallacy of arguing *post hoc ergo propter hoc, i.e.* of misinterpreting time-sequence in terms of causation, is by no means uncommon. Its absurdity appears in such an example as the following: "At twelve, noon, a 'hooter' sounded for one minute in a Clydebank shipyard, and immediately afterwards work on Waterloo Bridge, London, ceased for the men's dinner-hour."

(4) **Relations of Degree.**—Any quality which is capable of varying in degree, when we attend to its variations in different cases and compare them, may become a basis for inference under this head: "A is greater than B, and B is greater than C, therefore A is greater than C." The conclusion follows immediately from consideration of the combination of the relations. An argument of this kind proceeding from one degree to another through an intermediate degree, is said to be *a fortiori*. The application

of the principle is not limited to comparison of qualities capable of physical measurement, as the following example from Macaulay's speech on the " Dissenters' Chapels Bill " (1843) may show: " This is not the case of a possessor who has been for many years receiving great emoluments from land to which he had not a good title. It is the case of a possessor who has, from resources which were undoubtedly his own, expended on the land much more than it was originally worth. Even in the former case, it has been the policy of all wise law-givers to fix a time of limitation (for contesting the title); *a fortiori*, there ought to be a time of limitation in the latter case."

(5) **Relations of Number.**—Inference based on relations of number, when the numbers are definitely expressed, leads to arithmetical calculation. At bottom, all arithmetical calculations rest upon *counting*; but the logical character of the process of counting depends on the definition of " number," and this is not a simple matter. Here we can only say that, as a mental process, counting is the discrimination of one *unit* from another in order to combine them in the result of the counting. We attend only to the numerical relations between the units, whatever their nature or qualities may be.

Numerical relations may also be expressed indefinitely, by the use of what in English grammar are called " indefinite numeral adjectives "—" many," " most," " few," and the like. When the attempt is made to express a proposition, whose subject is qualified by an indefinite numeral, in one of the four traditional forms, we have to fall back on the indefinite " some," which simply sacrifices part of the meaning of the proposition. But propositions involving indefinite numerals can be treated logically without any sacrifice of meaning, and in certain cases provide a

basis for a valid non-syllogistic reasoning; thus, if we understand that "most" (or, "a majority") means "more than half," we may say, "If the majority of a public meeting voted for the first resolution, and a majority also voted for the second, then some who voted for the first must also have voted for the second." Such inferences are possible when the indefiniteness of the numeral adjective can be so far modified that we can compare its extent with that of the whole which is referred to.

We repeat, therefore, that such reasonings as those which we have briefly illustrated, cannot be expressed in strict syllogistic form. They contain no syllogistic middle term; and they involve propositions which differ in their essential logical nature from those propositions which ascribe attributes to subjects or which assert relations of inclusion or exclusion between classes.

12. Logical Value of the Syllogism

Our whole treatment of syllogistic reasoning in the present chapter is a sustained illustration and defence of the proposition that the syllogism is a form in which we actually do reason. In everyday thinking, speaking, or writing, our arguments are abbreviated; but when our conclusions are challenged, we usually attempt to state the premisses on which they are based, and in many cases—not in all—this statement takes the form of a syllogism. The syllogism therefore is an important form of deductive inference. We have also seen that it is not the only form of deductive inference: nor, indeed, is it the only important form.

We have now to deal with two further questions about syllogistic reasoning. These questions have been raised by certain logicians in ancient and modern times, and some

of these logicians have given answers adverse to the claims of the syllogism to be an important form of deductive reasoning.

(1) The first question is this: does the syllogism give us *new* knowledge? The question can only mean, does the syllogism give us knowledge which we did not possess before? Does anyone ever learn anything by means of syllogistic inference? This is equivalent to asking whether there is *genuine inference* in a syllogistic argument; since, unless we do learn something which we did not previously know, there is no inference. It may at once be granted that many examples of syllogisms given in textbooks on Logic are so familiar that the reader does not *acquire* fresh information from the combination of the premisses. For example, the reader of this book knew that "Socrates is a man" and that "all men are mortal" long before he found these premisses so put together as to constitute a syllogism in the first Figure. It would not be difficult, however, to find examples of premisses, both of which are known, but which had never been put together in the mind so as to yield a syllogism. It is a familiar fact that the minds of not a few individuals are divided as it were into "compartments" between which nothing can logically pass, so that opinions and beliefs belonging to one "compartment" are not brought into logical relation with those in another.

The conclusion of a deductive inference can never be entirely "new"; for that would mean "entirely unconnected with these premisses," so that it would not follow from these premisses, and there would be no inference at all. The conclusion is contained in the premisses *taken together;* it would offend against the rules of the syllogism if it told us anything not contained in the premisses.

When you have assumed the premisses you have assumed the conclusion. But when once we understand that the real act of inference consists in *the act of combining* the premisses, then we can understand that new knowledge can be obtained by the inference; where "new" knowledge means, not knowledge unconnected with the premisses, but knowledge which we did not have in our possession before. We have the new knowledge when we have put the premisses together, and not before; and we are able to obtain this new knowledge just because it is contained in the premisses when they are put together and because it could not be derived from either of the premisses separately.

(2) These considerations naturally lead up to the second of the two questions referred to above. Take the time-honoured example:—

All men are mortal;
Socrates is a man;
Therefore Socrates is mortal.

The question has been put in this way: *how do we know* that all men are mortal? If we know it only because of all the cases in which human beings are known to have died, then it is evident that *Socrates is one of these cases,* and the conclusion is simply part of the evidence on which the major premiss is based. Hence it has been said, the syllogism is not an argument to prove the conclusion, but a short formula reminding us that the conclusion is *contained in the major premiss:* "All these are so-and-so, and this is one of them."

Now we must carefully observe the kind of major premiss which is used in support of this opinion about the syllogism. The major is taken to be an **enumerative proposition.** An

" enumerative " proposition is a generalisation in which the " all " means merely " all cases hitherto observed." A completely enumerative proposition would be one in which we know that the cases observed are all the cases of the kind that are possible, so that there are no cases which we have not observed. This is only possible in the case of a definitely limited class, the limitations of which are known to us: " All the months of the year contain less than thirty-two days." Most enumerative propositions can refer only to " all cases observed." The opinion which we are discussing assumes that the major premiss is a generalisation enumerating a number of instances among which the denotation of the minor term is included. If so, the syllogism is not an argument but a " short formula " of the kind which we mentioned.

But the major premiss is not always, and is not usually, a merely enumerative proposition; and when the major premiss *is* an enumerative proposition, the minor term does not always, and does not usually, denote one of the instances included in the enumerative:—

All men are mortal;
The present Prime Minister is a man;
Therefore the present Prime Minister is mortal.

Here the minor term denotes an individual still living. He cannot be one of the cases on which the major premiss is based; and the conclusion is not " contained in the major premiss." We must again press the question, how do we know that " all men are mortal "? The answer is this: There are grounds—whatever account we may give of them—for ascribing attributes to certain *natures* or *kinds* of being, without going through the objects included under them or having any foreknowledge of their actual contents.

It is not necessary to know the natural history of all the varieties of mankind before we can venture to affirm mortality of human beings in general. The foresight of all its particular cases is not included in the meaning or in the evidence of a general rule; and a person may reasonably assent to the scientific statement of a natural law without any suspicion of the vast compass of facts over which its interpretation ranges.

A general proposition, therefore, is not necessarily an enumerative proposition referring merely to a collection of things. If we say "hemlock is poisonous," this does not mean that in certain cases I have seen it to be fatal; it means that investigation has established the fact that there is something in hemlock fatal to life. We may gather a general proposition *from a single instance,* provided that our investigation of it is sufficiently thorough. The conclusion of any typical syllogism, therefore, is not part of the evidence on which the major premiss is based; it cannot be derived from the major premiss alone but only from the two premisses in combination.

In those cases where the major premiss does express no more than a collective statement about a group of facts, and where the conclusion expresses one of these facts, we anticipate the conclusion in stating the major premiss. But even in such cases there is a genuine inference—a discovery of something not known from either premiss singly—*whenever we learn the two premisses at different times or by different means.* If I learn that the vessel XY was lost at sea with all on board, and learn subsequently, or by some other means, that my friend AB was a passenger on that vessel, then there is no doubt that the conclusion is "something new," although the major states a mere collective fact, which (for those who know, but not for me) already contains the conclusion.

The question discussed in this section has been dealt with by many writers since the first publication of John Stuart Mill's *System of Logic* in 1843. Most of them have dealt with it on the basis of a critical examination of Mill's analysis of the Syllogism. Mill argued (in his *Logic*, Bk. II., Ch. III.) that the syllogism, if taken as an argument to prove the conclusion, is a *petitio principii* or "begging of the question," because the conclusion is part of the evidence on which the major premiss rests (the essentials of Mill's argument, which is carelessly written, are summarised in Mellone's *Introductory Textbook of Logic*, Ch. VII., § 6). Some of the ancient Greek logicians stated the same doctrine much more clearly than Mill did; and the question was considered and answered by Aristotle himself (see the references given by Mellone, *op. cit.*, Ch. VII., note B., page 237). The most important discussion of the question, since Mill, is by W. E. Johnson, *Logic*, Vol. II., Introduction, page xix., and Ch I., § 3 (*cp.* Vol. I., pages 2, 3). For a short statement of Johnson's view see E. W. Whetnall's edition of Welton and Monahan, *An Intermediate Logic*, Ch. XX.

We will now work out some examples illustrative of conditional deductive reasoning. Each of the following arguments is to be expressed in strict logical form (so far as it is not already in the required form) and its validity is to be examined.

(1) "*If he succeeded, he must have been either very clever or very rich; but he was neither clever nor rich; hence he cannot have succeeded.*"

We must observe that a proposition is not disjunctive simply because it contains the words "either . . . or." This is a hypothetical proposition with alternatives in the "consequent." Both alternatives are denied in the minor premiss, and the conclusion is valid by denial of the consequent.

(2) "*Whenever a syllogism is valid, it contains not more than three terms; therefore this syllogism, which contains not more than three terms, is valid.*"

In logical form this is—" If a syllogism is valid it contains not more than three terms; the given syllogism contains

not more than three terms; therefore the given syllogism is valid." Here the minor premiss merely affirms the consequent of the major, and there can be no conclusion. The given syllogism may be invalid for other reasons. When the propositions are transformed into categoricals, the fallacy of "affirming the consequent" appears as "undistributed middle"; in this example, thus:—

All valid syllogisms are syllogisms which contain not more than three terms;
This syllogism is one which contains not more than three terms.

From these premisses there can be no conclusion, because the middle term is undistributed in both.

(3) "*If a man is a tyrant, he deserves to die; Caesar was not a tyrant, and therefore did not deserve to die.*"

Here the minor premiss merely denies the antecedent of the major, and so there can be no conclusion. When the propositions are transformed into categoricals, the fallacy of "denying the antecedent" appears as "illicit major"; in this example, thus:—

All tyrants are persons deserving to die;
Caesar is not one who was a tyrant;
Therefore Caesar is not one deserving to die.

The form of this syllogism is:—

All M is P;
No S is M;
∴ No S is P;

and the major term is distributed in the conclusion but not in the major premiss.

(4) "*If a man is educated he does not want to work with his hands; consequently, if education is universal, industry will cease.*"

The first question here is one of interpretation, so that the given statements may not be either obviously false or hopelessly vague. Some definite kind and degree of "education" must be referred to; otherwise the term may mean entirely different things in the two statements, and there is no logical connection between them. The major premiss therefore becomes: "If every man is educated in a stated way and to a stated extent, no man will want to work with his hands." The minor premiss then supposes, "All men are educated in that way and to that extent"; and the conclusion is, "No man wants to work with his hands." So stated, the argument is valid: but we must observe (*a*) it does not warrant the conclusion that all industry will cease; and (*b*) it refers only to some particular kind of education, not to all kinds of education.

(5) "*A man holds such opinions, if he is a Buddhist; therefore if you hold such opinions, you are a Buddhist.*"

Here the major premiss *is not* "If a man holds such opinions, he is a Buddhist"; but "If a man is a Buddhist, he holds such opinions." We may regard the hypothetical form of the minor premiss as rhetorical rather than logical. The minor merely affirms the consequent of the major, and there can be no conclusion.

(6) "*Giving advice is useless; for you either advise a man what he means to do, and the advice is superfluous; or you advise him what he does not mean to do, and the advice is ineffectual.*"

This is a "complex constructive" dilemma, and a fallacious one, because the minor premiss does not "exhaust the possibilities" (he may be undecided and your advice may move him in one way or the other): "If you advise a man what he means to do, the advice is superfluous, and if you advise him what he does not mean to do, the advice is ineffectual; but either you advise him what he does mean to do, or you advise him what he does not mean to do; therefore your advice is either superfluous or ineffectual." It can be "rebutted" by an equally fallacious dilemma of the same form, thus: "If you advise a man what he means to do, your advice is not ineffectual, and if you advise him what he does not mean to do, your advice is not superfluous; therefore (with the same minor premiss as before), the conclusion is: "Your advice is either not ineffectual or not superfluous."

(7) "M *is the only possible cause of* P; *hence, if we find* P *occurring, we may be sure of the presence of* M."

Causation is one of the relations which warrant a hypothetical proposition; thus, "M is the cause of P" warrants the proposition, "If M, then P." But here we are told that "M is *the only possible* cause of P," so that we may also say "If P, then M." And as P is given, we correctly infer M, by affirmation of the antecedent.

(8) "*If a man is rightfully entitled to the produce of his labour, then no one can be rightfully entitled to what is not the produce of his labour.*"

The major premiss is equivalent to: "If any man has produced this commodity by his own labour, he is rightfully entitled to the possession of it." The minor premiss is adequately represented by a single but entirely typical case: "This man has not produced this commodity by his

own labour." The minor merely denies the antecedent of the major, and there can be no conclusion.

(9) "*Only if you break the law are you imprisoned; that is why you are set free.*"

The major premiss is of the form "*only* if A, then B," *i.e.* A is the only antecedent of B" (compare Example (7) above); hence we may say, "if B, then A":—"If you are imprisoned, you are a law-breaker; you are not a law-breaker, therefore you are not imprisoned." Valid by denial of the consequent.

Exercise V

1. Which of the following arguments are logically correct?

   (*a*) A is B if it is C; it is not C; therefore it is not B.

   (*b*) A is not B, unless it is C; and as it is not C, it is not B.

   (*c*) If A is not B, C is not D; but as A is B, it follows that C is D.

   (*d*) A is not B if C is D, C, then, is not D, for A is B.

2. "If A is true, B is true; if B is true, C is true; if C is true, D is true." What is the effect upon the other assertions of supposing successively (*a*) that D is false; (*b*) that C is false; (*c*) that B is false; (*d*) that A is false?

3. "All M's that I have observed, together with all M's observed in past and present times by other persons, are P." Is this a true expression of the logical meaning of every general proposition? What is the bearing of this question on the nature of syllogistic reasoning?

4. State each of the following arguments in strict logical form, and examine its validity.

   (1) Restrictions on amusement should be avoided, for they are useless if not attended to, and injurious if they give rise to discontent.

   (2) This event could only happen at Rome, at Naples, or at Florence; it did not happen at Rome or at Florence; consequently it must have happened at Naples.

(3) If education is popular, compulsion is unnecessary; if unpopular, compulsion will not be tolerated.

(4) He maintains that nothing can be called virtuous unless it contributes to the welfare of man; he is therefore bound to maintain that every useful object, an article of food, for instance, is virtuous. (To be stated (*a*) as a hypothetical and (*b*) as a categorical syllogism.)

(5) Floods in the valley are due either to heavy rains or to the melting of snow. There has been neither of these recently, so there will be no flood.

(6) If all philosophical theories were sound, some would be accepted by a majority of thinkers; but since none are accepted by a majority of thinkers, none are sound.

(7) If Mr. A. is not in time, he will not be able to perform that ceremony, but we know that he is always in time, so he will be able to perform the ceremony.

(8) Darwin must have been very unhappy. For he said that he would feel very happy if he had only to observe, and not to write; and we know, of course, that he wrote many books.

(9) If observation and imagination are essential to sympathy, Mr. B. has a sympathetic disposition, for he possesses those qualities in an eminent degree.

(10) Agriculturists must be heavy losers; for if the summer is dry, they lose through the withering of the crops, and, if the summer is wet, they lose because the crops do not ripen.

(11) If, and only if, a tax is productive is it legitimate; and as a tax on windows is no longer regarded as legitimate, we may conclude that it is not productive.

(12) If the law were really impartial and punished "blasphemy" because it offends the feelings of believers, then it ought also to punish such preaching as offends the feelings of unbelievers; but the law imposes no restraints on the believer, however offensive his teaching may be to those who do not agree with him.

CHAPTER VII

INDUCTIVE REASONING

In the previous chapter we had occasion to discuss the opinion that the conclusion of any typical syllogism is part of the evidence on which the major premiss is based. In rejecting this view we called attention to some of the ways in which a general proposition, such as may be used for the major premiss of a syllogism, may be obtained. Sometimes such propositions are obtained by inference from other general propositions: but at bottom, or "in the last resort," they are obtained as inferences from observed facts.

This introduces us to the kind of reasoning called *inductive*, and to the difference between "induction" and "deduction." In deductive reasoning we start with our premisses as given. In the broadest sense, deductive reasoning is finding a place for some fact as a detail within a system (Chapter VI., page 105), when the general nature or character of the system is known beforehand. In this sense, we argue "from the general to the particular." Thus, in the special case of the syllogism, the general principle or rule which must form one of the premisses in all syllogistic reasoning, is taken as granted, for the purpose in view. We may express this metaphorically by saying that in deductive reasoning we start having in our hands the thread which unites the various facts. In inductive reasoning, on the other hand, we have to find the common thread. We start with certain kinds of facts which occur

together in our experience. We assume that there is some principle which unites them; and our object is to read out of their particular details the general law of their connection, and, if possible, to "explain" this connection by linking it up with other laws. This is to combine facts and laws into a systematic whole. We want to *organise* facts which, as they are given to us, appear to be isolated, fragmentary, and discrepant. The methods by which scientific generalisations are obtained from observed facts can be studied separately; the study of them is the principal problem of what is called "Methodology," or the "Theory of Scientific Method," or, more commonly, "Inductive Logic."

1. Elementary Forms of Induction

It may seem as if inductive reasonings, described in the general terms which have employed above, were far removed from the reasonings of common life. But it is not only in scientific matters that we employ inductive methods. In the commonest affairs we are continually seeking to explain or account for what happens, or analysing a practical situation or practical difficulty in the light of previous knowledge and experience. In so doing we employ, in an elementary form, the genuine method of science. Science has been described as "organised common sense." We now proceed to illustrate some of the elementary forms of inductive reasoning in familiar cases (see also Chapter I., pages 10, 11).

Professor John Dewey, quoting Wordsworth's pregnant saying—

> The eye, it cannot choose but see;
> We cannot bid the ear be still;
> Our bodies feel, where'er they be,
> Against or with our will—

observed that this holds good in the degree in which we are naturally possessed by curiosity. An inert mind waits, as it were, for experiences to be forced upon it. The curious mind is constantly alert and exploring, seeking material for thought, and is eager for new and varied experience. It carries us far beyond what the practical situations or difficulties of life demand. Curiosity at length becomes *wonder*—the only guarantee of the acquisition of the primary facts on which inference must base itself.

When we look not at its final issues but at its first beginnings, curiosity is scarcely more than an expression of abundant bodily energy. Observers of *animals* have noted what Professor Hobhouse has called " their inveterate tendency to *fool* " : " rats run about, smell, dig, or gnaw without real reference to the business in hand; the dog scrabbles and jumps, the kitten wanders and picks, the otter slips about everywhere like ground-lightning, the elephant fumbles ceaselessly, the monkey pulls things about." The activities of a young child reveal a ceaseless display of exploring and testing. Objects are fingered and handled, drawn and pushed and thrown, until (it appears) they cease to yield new qualities.

When the child learns that he can appeal to others to enlarge his store of experiences, a new stage is reached. He asks in succession what holds up the house, what holds up the soil that holds the house, what holds up the earth that holds the soil. " But his questions," says Dewey, " are not evidence of any genuine consciousness of rational connections; his ' why ' is not a demand for scientific explanation; the motive behind it is simply eagerness for a larger acquaintance with the mysterious world in which he is placed. The search is not for a law or a principle,

but only for a larger fact." Yet there is more than a desire to accumulate mere information or to heap up merely disconnected items, although sometimes the interrogating habit threatens to degenerate into a disease of language. In the feeling, however dim, that the facts which directly meet the senses are not the whole story, that there is more behind them and more to come from them, lies the germ of *intellectual* curiosity.

Curiosity becomes intellectual when it takes the form of purposive and systematic interest in *questions or problems* set by the observation of things and the accumulation of experience. To the alert and open mind, nature and social experience are full of varied challenges to *inquire further.*

Among the Greeks, the capacity for knowledge—not merely knowledge as information but as the rational understanding of the causes and consequences of things—was believed without question to be man's highest endowment, and its development to be at once a duty and a delight. Plato expressed this conviction in his own way, when he said, "It is a happy genealogy which makes *Iris* the daughter of *Thaumas*," *i e.* which treats the messenger of the gods, the winged thought which passes to and fro between heaven and earth, as the child of Wonder, the impulse to know and understand. Aristotle believed that the highest life is a life of truth-seeking and truth-seeing, ever successful, yet perennially interesting.

"In a few people," says Dewey again, "intellectual curiosity is so insatiable that nothing will discourage it, but in most its edge is easily dulled and blunted. Bacon's saying that we must become as little children in order to enter the kingdom of science, is at once a reminder of the open-minded wonder of childhood, and of the ease with which this endowment is lost."

We shall now illustrate some of the elementary forms of induction in common life, beginning with a simple practical situation. Imagine a man walking on a warm day, his attention being mainly occupied with things other than

the weather. Suddenly he notices that the air is cooler. It occurs to him that a shower is possible. Looking up, he sees a dark cloud between himself and the sun; and he then quickens his steps. The possibility of rain is something suggested by association; he *perceives* the changes in the temperature and the sky, and he thinks of a coming shower. What are the sources of the suggestion? Its sources are in past experience and present knowledge as affected by that experience. He is acquainted with similar situations, and has dealt with material of the same kind before. The present fact understood through past experience and knowledge, forms the ground for an inductive conclusion.

Let us take another illustration, the subject of which would be familiar, a few years ago, to people travelling in Brooklyn and New York. In this case the problem is not merely a practical one, but appeals to a more theoretic interest. "Projecting nearly horizontally from the upper deck of the ferry-boat on which I daily cross the river, is a long white pole with a gilded ball at its tip. (*a*) Its colour, shape, and gilded ball suggested a flag-staff when I first saw it. But the pole was nearly horizontal, an unusual position for a flag-staff; there was no pully or cord by which to attach a flag; and elsewhere on the boat were two vertical poles from which flags were occasionally flown. I then thought of other purposes for such a pole. (*b*) Was it an ornament? But not only other ferry-boats but even tug-boats carried such poles. (*c*) Was it in connection with a wireless apparatus? But its position, and the absence of any visible fittings for such a purpose, made this improbable. (*d*) Its purpose might be to point out the direction in which the boat was moving. The pilot's position was near the fore-part of the boat, so that he would need some such guide as to its direction. The pole

was lower than the pilot-house, so that the pilot could see the whole of it; and its tip was so far higher than its base, that it appeared to him to project out in front of the boat. Tug-boats would also need poles for such a purpose. This hypothesis was so much more probable than the others that I accepted it." The reader will find, on consideration, that each of the suggestions marked (*a*), (*b*), and (*c*) above is negatived in a "mixed" hypothetical syllogism valid by denial of the consequent. The acceptance of suggestion (*d*) is based on four hypothetical syllogisms, the character of which will be pointed out below.

In this piece of reflective thinking, four steps can be discerned.

(1) The perception, and then the more precise definition, of a difficulty, problem, or question. One of the essential characteristics of scientific thinking, and of all thinking animated by the scientific spirit, is to grasp the nature of the problem before proceeding to attempts at its solution.

(2) The suggestion of an explanation or possible solution —a "conjecture," "supposition," or "hypothesis," suggested because the facts are understood in the light of previous experience. In the example given, four different suppositions were tried.

(3) The development of the bearings or implications of the supposition or conjecture. This part of the inquiry is essentially *deductive*; when expressed in strict logical form becomes a hypothetical syllogism.

(4) The concluding and (sometimes) the conclusive step is the verification of the conjecture by observation or experiment. This has been very clearly stated by Dewey: "Reasoning shows that *if* the conjecture be adopted,

certain consequences follow. So far the conclusion is hypothetical or conditional. If we look and find present all the conditions demanded by the theory, and if we find the characteristic traits called for by rival alternatives to be lacking, the tendency to believe, to accept, is almost irresistible. Sometimes direct observation furnishes corroboration, as in the case of the pole on the boat. In other cases experiment is required; that is, conditions are deliberately arranged in accord with the requirements of an idea or hypothesis to see if the results theoretically indicated by the idea actually occur. If it is found that the experimental results agree with the rationally deduced results, and *if there is reason to believe that only the conditions in question would yield such results*, the confirmation is so strong as to induce a conclusion—at least until contrary facts shall indicate the advisability of its revision."

The sentence italicised in this statement calls attention to a condition which is not usually fulfilled in the case of the ordinary hypothetical proposition, but which, when it is fulfilled, enables us to argue from the affirmation of the consequent. The condition is this. We must know not merely that A is an antecedent of B, but that A is the *only possible antecedent* of B (a symbolic example of this condition has already been given, see page 186). In this case we may affirm "If A, then B," *and* "If B, then A." Moreover another important consideration arises in this connection. Although the hypothetical propositions which occur in our actual reflective thinking are not usually of this "reciprocal" type ("If A, then B" *and* "If B, then A"), yet, when the premisses of a hypothetical syllogism are true, a conclusion from the affirmative of the consequent, though technically invalid, may suggest a possibility or even a probability. And when we have a number of

independent syllogisms of this kind, all pointing to the same conclusion, the probability of the conclusion becomes very high indeed. Thus, the conclusion marked (*d*) in the example given above, is accepted on the ground of four independent hypothetical syllogisms, each technically invalid (by affirmation of the consequent), but all pointing to the same conclusion; and no other probable explanation can be found.

The reader will observe that in the example which we have given there is an upward and downward movement, from the facts to conjectural explanation (inductively) and from conjectural explanation back to the facts (deductively) —back and forth, as it were, between fact and interpretation. The idea first suggested (inductively) is employed to reason out (hypothetically) certain additional particulars not yet experienced, which ought to be there if the suggestion is correct.

Scientific induction is an elaboration of the process which, in a comparatively simple form, we have illustrated. Science demands a higher degree of exactness and thoroughness. This elaboration brings about specialisation, a marking-off of various types of problems from one another, and a corresponding classification of the materials of experience associated with each type of problem.[1]

2. The Causal Relation

In the inductive reasonings of common life, the explanatory suggestions which occur to a person in a given situation depend, very largely, upon the prevalent direction of his interests, the general character of his past experiences, his special training, the things which had

[1] For some of the examples in this Section, I am indebted to Professor John Dewey's suggestive book *How We Think* (new edition, 1933).

recently occupied his mind, and so forth. We see the same thing in science, with the difference that the observer's experience is more thorough and more specialised. A scientific hypothesis is the offspring of a prepared or trained or experienced mind.

"Modern discoveries," says De Morgan, "have not been made by large collections of facts with subsequent discussion, separation, and resulting deduction of a truth thus rendered perceptible. A few facts have suggested a *hypothesis*, which means a *supposition* proper to explain them. The necessary results of this supposition are worked out, and then . . . other facts are examined to see if these results (of the supposition) are found in nature." Hypotheses are suggested, he adds, "not by rule, but by a sagacity of which no description can be given, precisely because the very owners of it do not act under laws perceptible to themselves." Nature, herself, as we shall see, does give broad hints as to the direction in which a fruitful hypothesis may be looked for; but only a prepared mind knows how to "take the hint."

Now there is an extensive field of inductive investigation where the problems are comparatively simple, in this sense: the hypothesis is simply the **suggestion of a possible cause** of some event in which we are interested, and no deductive reasoning from the hypothesis is necessary. The suggested causal relation lies open to observation or can be put to the test of experiment. In all such cases the verification of the hypothesis is usually described as being by **direct induction.**

The next question therefore is this: What do we mean by "cause"? Two questions are involved. (*a*) What is the general meaning of the word in the vaguer usage of ordinary speech? (*b*) What meaning of the word is most convenient and suitable for use in scientific investigation?

The use of the word "cause" as a concrete general term or "common noun," though to a great extent unavoidable, has very misleading associations. We speak of "causes" as though they were natural objects like trees, stones, etc.; and the abstract terms "causation," "causality," are used as if they were names of some kind of power which the "cause" possesses and which enables it to produce "effects." All this, of course, is mere mythology. A "cause" is something that happens; it is an *event*. When we single out some particular causal event for special attention or investigation, we call it "*a* cause"; and when we speak of "causation" or "causality," we think of the causal event as part of a process with antecedents and consequents. But the terms "causation" and "causality" are best abandoned in favour of the more accurate term **causal relation.** In any case we must always think of a cause not as a "thing" but as an *event*.

We must observe that, in their most general meaning, "antecedents" and "consequents" are not necessarily "causes" and "effects"; in other words, the relation of antecedent and consequent has a wider meaning than the causal relation. In its most general meaning, an antecedent is any circumstance, condition, or event which exists before some other event which is different from the antecedent and which we distinguish from it. But *not all antecedents are causes*. The sun's light may be an antecedent to the burning of a house, but not the cause, for the house would burn equally well in the night. In fact, the identification of any mere *antecedent* with the *cause* may lead to absurd conclusions. On the other hand, when we have found an antecedent which is *indispensable or necessary for the occurrence of an event*, we have found a *cause* of the event, which we may then regard as its *effect*.

Now there are usually many different circumstances necessary for the production of an effect, and, in the wider sense of the term, all of them must be considered causes or necessary parts of the cause. "Thus," it has been said, "the cause of the explosion in a rifle is not simply the pulling of the trigger, which is only the last apparent cause or *occasion* of the explosion; the qualities of the powder, the proper form of the barrel, the proper construction of the cartridge, the existence of the surrounding atmosphere, are among the circumstances necessary to the loud report of the rifle; any of them being absent, it would not have occurred." What is here called the "occasion" of the effect, we shall call the "immediate cause"; the other circumstances present and necessary for the effect we shall call "causal conditions," or more briefly "conditions." It is by no means always easy, in any given case, to say what *is* the "immediate cause." We shall see that special methods have to be devised for dealing scientifically with this problem.

The term "negative condition" has also come into use, and leads to no difficulty if accurately defined "The negative conditions," says Mill, "may be all summed up under one head, namely, the absence of preventing or counteracting causes." "By a negative condition of any result," says Professor A. Wolf, "is meant the absence of whatever may thwart the appearance of that result; while other conditions, the so-called positive conditions, may be said to contribute positively to the result. Thus, for example, the velocity of falling bodies is conditioned positively by the *time* of their fall, not by their *mass* Two bodies of different mass or density, say snowflakes and hailstones, should, therefore fall through the same distance in equal times; but that will not be the case if they fall through a resisting medium such as the air. The absence of a resisting medium is therefore the *negative condition* of the result contemplated, namely, the fall of different bodies, having different masses or densities, through equal distances in equal times" (*Textbook of Logic*, Ch. XXIII., § 3).

Take again a case of an effect with antecedent *conditions*. A person eats of a certain dish and dies "in consequence"

—that is, would not have died if he had not eaten of it. Not only the food, but the taking of it in combination with a particular constitution, state of health, climate, etc.—these constitute the group of conditions from which the fatal result follows. This is clearly seen in cases where two persons who have had an identical experience react to it differently. What kills one merely makes another ill.

In popular thought, or in everyday life, we choose one of these conditions and call it, in a special way, *the* cause. Usually, in popular thinking, we select it somewhat arbitrarily, or for some special practical purpose. We do not usually go beyond the preceding circumstances out of which "the cause" arose—*e.g.* in the case of one shot through the heart, we take as cause the action of the person who fired the bullet. Such antecedent circumstances as are striking and important from some practical point of view, are the "causes" with which we concern ourselves. Sometimes, what is practically the most important is scientifically the least important. It may be of great practical importance to know what circumstances will produce a result without knowing *how* they produce it. For instance, it may be of importance to clear the premises of rats; traps, strychnine, phosphorus, and terriers are various "causes" between which we may choose: but we do not hold *post-mortems* on dead rats.

For scientific purposes, it is evident that greater precision is required. Science seeks for the "immediate cause." We have already introduced this term. For scientific purposes it must be understood definitely and in its literal meaning. The terms of a causal relation stand, not for two *separate* events, but for two events *distinct in time.* Cause and effect are divided simply by a mathematical line—a line destitute of breadth—which is thrown by

our thought " across " and " along " the current of events. There is no pause in reality; the whole process is continuous; the immediate cause comes into full action only at the very moment when the effect begins to be produced. Thus, the entrance of " microbes " into a human body is followed by a certain disease: this is true; but the essential factor is, that as soon as the microbes effect a lodgment in the human body they begin to secrete injurious substances. In Chemistry, again, the union of oxygen and hydrogen in the proportion by weight of eight to one is not an event separate from the formation of water. The whole process, we repeat, is continuous; and in this continuous process we can distinguish particular events and investigate their relations.

When, in scientific investigation, we select from the concurrent conditions one, which we call the " immediate cause," the selection is in no sense arbitrary. It is made according to strict methods which we shall endeavour to analyse in the sequel. But before proceeding to this, there is another important question arising from our account of the causal relation. Suppose that in some particular case we have established the fact that an event E is the effect of A. This fact does not warrant the conclusion that A is the *only possible cause* of E. There may be other different causes, B, C, D, each capable of producing the same effect E. And if so, though we can argue from cause to effect, " if A, then E," " if B, then E," and so forth; we cannot argue from effect to cause, " if E, then A."

When this appears to occur, we have what is called a **plurality of causes.** By this is *not* meant that many different or distinguishable conditions may *co-operate* in the production of an effect. The doctrine of " plurality of causes " means that the same effect may be due sometimes to one cause, sometimes to another. Now there is no

doubt that in many cases where there appears to be a plurality of causes, *the "plurality" disappears before a more exact scientific investigation.* Let us consider some examples.

"There are many causes of motion"—visible impact; heat; electrical and magnetic action; gravitation. Yet the doctrine of the Conservation of Energy, which rules modern Physics, means practically that all motion in matter is produced in the same way, namely, by other motions in matter. "There are many causes of death." But life is a complex process consisting of a multitude of co-operating processes, of which some are directly essential. If any one of these essential processes is interfered with, life ceases; and the interference can only be of one kind. Hence there are many causes of death only because there are many kinds of death; "death" is a fact as complex as "life." Again: "A disease may have many different causes." But the youngest and most successful of recent scientific studies—sometimes called *Bacteriology*—has proved beyond doubt that each kind of disease—among those most inimical to life—is produced by the entrance into the human body of one particular kind of the extremely minute living organisms known as "microbes." Thus, when the apparent "many causes" of the disease are analysed, there is found to be something fundamental, common to them all—namely, the presence of these minute forms of life. Each disease has its characteristic "microbe."

The doctrine of plurality is only a practical working caution. In the absence of knowledge of the immediate cause, we have to bear in mind that different combinations of circumstances may bring about the same event. Practically we have to begin the investigation by examining those different combinations of circumstances in which the

event is produced—considering them, at first, as so many different "causes." They are not the immediate cause; but it is *operative in them.* As a practical caution, "plurality of causes" is equivalent to the rule which forbids argument from the negation of the antecedent or the affirmation of the consequent in a hypothetical proposition.

The reader will observe that we have not denied the possibility of any "plurality of causes" in the nature of things. Such a denial would be mere dogmatism. We have only said that in many cases, where there appears to be a plurality of causes, the plurality disappears before a more exact scientific investigation. And whenever the plurality of causes thus disappears, it is because the investigation has established a *reciprocal* causal relation, which warrants the two propositions, "If A, then E," *and* "If E, then A"; or, which is the same thing, "If A, then E," *and* "If not A, then not E." This is what we mean by saying that A is the "immediate cause" of E.

Strictly speaking, we cannot stop at *any limited* combination of conditions and say "these and nothing else are responsible for the production of the effect"; for all events are connected together—when a stone is dropped, there is a sense in which it has an effect through all time and all space. In reality, any event is the effect of all causes operative throughout the universe. But all these further and more remote conditions are usually taken for granted—in fact, necessarily so. What we want to know is the immediate cause. The scientific investigator seeks to isolate the event in various ways, and examine it under conditions which if possible he arranges for himself, so as to discover among these conditions some definite circumstance *with which the event will occur and without which it will not occur.*

The immediate cause is but one out of a group of conditions necessary for the effect; but sometimes it is more convenient to regard the immediate cause itself as a group of facts acting together rather than as a single fact. For example, in the formation of water by the passage of an electric spark through a vessel containing two parts (by volume) of Hydrogen and one of Oxygen, the immediate cause is the one fact of the action of the electric energy, whatever it may be. On the other hand, in the investigation of the theory of Evolution, it is more convenient to consider the possibility of several different immediate causes of the modification of species, *e.g.* "natural selection"; the direct action of the environment; the inheritance of characteristics acquired by the creatures' own activities, etc. Again, in the case of a person's death through being shot in the heart, the immediate cause is the piercing of the heart by the bullet, which we may regard as a single fact. This stops the heart's action; and the heart's action is one of the processes necessary in order that the complex process of physical life should continue.

3. Laws of Nature

When investigation has established a causal relation which can be stated in the form of a hypothetical proposition, "If A, then E," this is equivalent to the discovery of a Law of Nature. We say "discovery," because the relation in question is not invented or created by the investigator. Science assumes that natural laws are *objective*—that is, independent of the investigator or discoverer. But the investigator desires to give as accurate a statement of the relation as possible—he puts it into the shape of a "formula," which may or may not need correction afterwards. The term "Law of Nature" is

applied both to the objective relation itself and to the man-made formula. The two meanings must not be confused. We may combine them by saying that a Law of Nature is *a general statement of a constant relation between a particular kind of cause and a particular kind of effect.* When formulated as a hypothetical proposition, the antecedent expresses the cause and the consequent the effect. The following may be given as elementary examples: " If a material body is heated, its volume changes "; " If a pendulum is lengthened, its oscillations widen." Laws of Nature are Laws of Causation.

Now, if we are to be able to formulate any laws of causation at all, we must make a very important assumption about the actual structure of the objective world. This assumption is usually called the **Law of the Uniformity of Nature,** and placed on the same level with a second fundamental assumption, called the **Law of Universal Causation.**

Take the latter first. If scientific investigation is to be possible, we must grant that *every event has a cause.* This is a necessary pre-supposition or postulate of scientific method; but not only so. It is involved in all rational thinking. Even children, and the lower races of men, if they do not think *of* it, think *according to* it. If the " savage " were content to leave any event unexplained, he would not imagine that so many events are controlled by spirits, beneficent or the reverse. It is in fact impossible to think of an event without referring it to a cause known or unknown. The Law of Uniformity may be formulated in the statement that *the same cause will have the same effect;* when the same conditions are fulfilled, the same result will follow. The reader will see, on reflection, that this principle is *included in* the principle of universal causation; for by " cause " is at least meant a condition

on which the effect *always* follows; if it sometimes followed and sometimes did not, there would be no object in trying to discover the cause; there would be no real *cause* to discover.

The question of "exceptions" to the uniformity of Nature has sometimes been raised: are we "certain," for instance, that the sun will rise to-morrow morning? This question is not about *causation* at all; it is about the permanence of the present arrangements in Nature. Experience, quite apart from science, shows that there are general kinds of orderly succession in the outward course of events; such as appear in the succession of day and night, summer and winter, seed-time and harvest, life and death. The regular succession of events in a thousand different ways accustoms us, from force of habit, to expect things to happen in the regular order in which we have experienced them; and we find that the expectation is fulfilled. This constitutes an overwhelming presumption in favour of the maintenance of the present arrangements in Nature, but it does not prove that deviations from this order are impossible. An expectation, bred by experience and custom, that events will occur in a certain way is not the same as a knowledge that they must so occur; and such knowledge is not in our possession. We have no grounds for affirming that the sun absolutely *must* rise to-morrow morning; there is only an overwhelming presumption in favour of the expectation that it will. In this sense we may say that there are uniformities or regularities in Nature which are *not causal.*

We do not regard night as the cause of day; and the seasons are not conceived as producing one another. Their regular succession is part of Nature's routine, and custom has led us to expect its continuance. The times of the

sun's rising and setting are foretold for many months in advance: on the assumption that the earth's motions, relatively to the sun, will continue as they are now known to be, and that no new force will cause any appreciable disturbance. In like manner there is a presumption—though we cannot call it "overwhelming"—against the occurrence of an entirely new departure in Nature; but the presumption from past experience would have to give way before sufficiently strong evidence, if such were produced. We should have to believe even the most extraordinary story if the evidence on which it rested were impregnable, though *the more extraordinary the story, the more unusually strong would the evidence for it have to be.*

We may pursue this latter supposition a little further. Let us suppose that some event, so extraordinary as to suggest an entirely new departure in Nature, were proved to have occurred on a certain date. Now we know that a Law of Nature can be expressed in the form of a supposition or hypothetical statement. wherever and whenever the same cause occurs, the same effect will follow. The law only "comes into effect" when the causes actually occur in the series of events in space and time. But it is conceivable that the cause might only so occur once in a thousand or in a million years; and then the law would only come into operation once in all that time.

An event might happen, the like of which had never been heard of within the memory of man, and yet involve no breach of any Law of Uniform Causation. So to regard it, would be the instinct of scientific observers in such a case. They would make the most accurate record of the event in all its circumstances, and preserve it in the hope that subsequent discovery might throw light upon it, or that, at some time, it might happen again They would regard it as due to the operation of some law or laws hitherto utterly unknown.

In addition to the uniformities of sequence, with which we have been concerned in the preceding paragraphs, Nature shows us numerous **Uniformities of Co-existence.** By "uniformities of co-existence" we mean cases where two distinguishable attributes (we speak of *two* only for

simplicity's sake) are always found conjoined throughout some particular class or kind of being. Now sometimes it can be ascertained that one of these attributes is indispensable to the existence of the other (*i.e.* they constitute a causal relation without sequence). Sometimes—more often, in fact—they are found to be *joint-effects* of previous causes. Many cases of uniform co-existence can be so explained. The distribution and arrangement of material objects throughout the universe, are all the results of causation, starting from previous conditions. The distribution of sea and land, the stratification of the earth's crust, the existence of an atmosphere, the distribution of the materials of the globe generally—are the result of natural agencies or forces, operating upon previous conditions. Some kinds of uniform co-existence are very conspicuous in Nature. Such are the several kinds of inorganic matter known in Chemistry as "elements"; the many classes of minerals as set forth in Mineralogy; and, above all, the structure and characteristic properties of the different kinds of animal and vegetable life. It is on such uniformities of co-existence that *classification* is based. All classification implies a previous observation of the fact that certain attributes always accompany one another (see Chapter III., page 54).

The question, how far each of these kinds of "co-existence" can be causally explained, is one for the various special sciences to investigate. But the significant fact remains, thus stated by John Stuart Mill: the same considerations, which compel us to recognise that many uniformities of co-existence are capable of causal explanation, "compel us also to recognise that there must be one class of co-existences which cannot depend on causation. the co-existences between the ultimate properties of things—those properties which are the ultimate causes of all phenomena, but are not themselves caused by any phenomenon, and a cause for which could only be sought by ascending to the origin of all things." (*Logic*, Bk. III., Ch. XXII., § 2.)

4. Hypotheses of Immediate Causation

We now come to the all-important question: how Laws are discovered and proved. The first step is the suggestion of a possible immediate cause of the event which is under investigation; in other words, a hypothesis of immediate causation. There are, then, two questions: How came the inquirer to think of this suggestion as a real possibility worth testing? And how, once suggested, is it to be tested, and, when valid, proved?

We must repeat that no possible code of rules can be laid down for the creation of scientific hypotheses. Such a hypothesis is the offspring of a prepared mind. But we said that Nature sometimes gives "broad hints" to the prepared mind; or, to vary the metaphor, she erects "finger-posts" pointing out lines of fruitful inquiry. We proceed to discuss two important ways in which Nature does this. These are **enumeration** and **analogy.**

(1) The term "enumeration" (or "simple enumeration"), as a technical term in Logic, retains its ordinary meaning of "counting," but it means also that the counting of particular things is made the basis of a generalisation. The enumeration may be "complete" or "incomplete." It is *complete* when we know that we have gone over all the particular cases, and, finding that each one has (for instance) the quality P, we conclude that they all are P. Thus we conclude by complete enumeration that "All the months of the year have less than thirty-two days"; for the number of months is limited, and we can ascertain the fact in each particular case before making the general statement. The enumeration is *incomplete* when (as usually happens) we cannot go over all the particular cases, for some of them may be in the future or be otherwise inaccessible to us. Then—to take an example which is not so

trivial as it appears—we may conclude in such a way as the following: "This cat has a tail, and that one, and that one, up to all that I have seen or heard of; therefore all cats (without exception) have tails." An incomplete enumeration is a generalisation in which the "all" means "all cases hitherto observed" and is then extended to all cases without exception.

What we have called "complete enumeration" was formerly called **perfect induction,** and "incomplete enumeration" was called **imperfect induction.** These terms are nearly obsolete; but if the reader meets with them, they are to be understood as here defined.

It has been said that a conclusion based on complete enumeration ("perfect induction") is of no scientific value whatever, because the conclusion is only a re-assertion in briefer form of the premisses. To this, Jevons has well replied: "If 'perfect induction' were no more than a process of abbreviation, it is yet of great importance, and requires to be continually used in science and common life. Without it we could never make a comprehensive statement, but should be obliged to enumerate every particular. After examining the books in a library and finding them to be all English books, we should be unable to sum up our results in the one proposition, 'all the books in this library are English books'; but should be required to go over the list of books every time we desired to make any one acquainted with the contents of the library. The fact is, that the power of expressing a great number of particular facts in a very brief space is essential to the progress of science. Just as the whole art of arithmetic consists in nothing but a series of processes for abbreviating addition and subtraction, and enabling us to deal with a great number of units in a very short time, so 'perfect induction'

is absolutely necessary to enable us to deal with a great number of particular facts in a very brief space."

A conclusion based on incomplete enumeration is in a different position. A single negative instance will refute it. As regards the example given, the typical Manx cat together with some species in other parts of the world are found to be tail-less. No mere counting of instances will make a conclusion more certain. We may know that S and P are conjoined twice or two thousand or two million times; but this does not warrant us in saying that they are *always conjoined* unless we have something more than the mere number to go upon. But for the very reason that incomplete enumeration can never provide *proof*, it may point to a fruitful line of inquiry suggesting a hypothesis capable of scientific investigation. The very fact that S and P are found to be conjoined in so many cases forcibly suggests an examination of the cases to see if they agree in any other material circumstance except SP. If they do not, then a real connection of S and P is suggested for further examination and testing.

(2) Hypotheses of immediate causation are also frequently suggested by Analogy. In order to show how this arises, we must explain the logical nature of an argument from Analogy. We must distinguish (*a*) an Analogy, and (*b*) an argument from (or based upon) an Analogy. A genuine Analogy is an *important resemblance* between two objects or events; and by "important" we mean a resemblance in some quality of a fundamental or essential character. An argument from the Analogy is an inference that because a certain proposition is true of the one case it will also be true of the other. The inference varies in value according to the importance of the resemblance on which it is based. When the resemblance consists in merely

"accidental" qualities, then there is no genuine Analogy at all, and arguments based on such resemblances are worthless.

On what does the *importance* of the points of resemblance depend? It does not depend on the mere number of the resemblances. Two cases may resemble one another in a very large number of unimportant respects, affording not the least ground for inferring a resemblance in any other quality. For instance, two boys may resemble one another in height, features, strength, and other physical gifts, may be of the same age, born in the same town, educated in the same way, come from families of similar social position and cultivation; yet could we infer that because one of them has native mental abilities of a high order, the other will have the same? If the *number* of points of resemblance were the essential thing, the argument ought to possess some force; but it is clearly worthless. The reason is that none of the points of resemblance are fundamental or essential. To provide a strong argument from Analogy, the *resemblances must be essential and the differences unessential.* General experience, and sufficient knowledge of the subject to which the given Analogy belongs, are the only means of distinguishing the essential from the unessential.

In analogical inference, then, a new case is shown to be probably an instance of an event whose cause is known to be illustrated in a case with which we are familiar. It is an argument **from particular to particular,** from one instance or example to another (see Chapter I., page 13) depending on the resemblance between the two cases in some material circumstance. But it inevitably suggests that both cases may be instances of a general law under which they fall. It prompts us to extend our knowledge of the first case and found on it a law of connection which

includes the second. Thus, suppose we have a suggestion, however it may be made, that S and P are really connected. If we find some relation MR to be an important factor *in* S, and M to be also an important factor in P, then the argument from Analogy is, that P as well as S is MR. Then we may support the original suggestion by an inference as follows:—

P is MR,
S is MR;

therefore S and P are probably causally connected through MR. The fact that the premisses are those of an invalid syllogism in Figure II simply means that no argument from Analogy can ever be entirely conclusive. The hypothesis suggested by the premisses must be tested by further examination of the connection of MR with S and with P.

For concrete examples, the reader may refer to our discussion of the connection between "falling smoke" and "rain" (Chapter I., page 11), which consists of a suggestion based on enumerative agreement, followed by an analogical justification of the same suggestion, leading to an explanation of the suggestion by a law of real causation.

We must add that although no single argument from Analogy can be conclusive, we may have a convergence of analogical arguments leading to practical certainty; thus:—

(*a*) In districts of the earth now exposed to glacial action we find scored or "striated" rocks;
In such and such a valley in Great Britain we find striated rocks;
Therefore this valley probably has been exposed to glacial action.

(*b*) In districts now exposed to glacial action we find perched boulders;
In the same valley we find perched boulders;
Therefore this valley probably has been exposed to glacial action.

(*c*) In districts now exposed to glacial action we find lateral and terminal " moraines ";
In the same valley we find lateral and terminal moraines;
Therefore this valley probably has been exposed to glacial action.

Such a convergence of analogies, each inconclusive if taken by itself, leaves no room for doubt. Of one such case, Darwin said: " A house burnt down by fire did not tell its story more plainly than did this valley."

We have defined an Analogy as a resemblance between two cases in some really important characteristics, and an argument from Analogy as an inference from the one case to the other, based on this resemblance. Unfortunately there is much vagueness, and even confusion, in the current use of the word. Occasionally we find it used for *any kind* of resemblance between two or more objects or events. Such a resemblance may be seized upon by the mind as a justification for the use of a " metaphor." It may appeal to the imagination, and so be useful in the way of illustration. It may be used by the literary artist with power and beauty, as in Shelley's " Skylark." In the metaphors of literature and art there is of course no question of any logical argument from one case to another; and the use of the word Analogy in this connection is entirely unjustifiable.

Fallacious analogical inferences have played a great part in the history of superstition. The primitive man believes

that all things are "animated" because they move, and because he sees them in his dreams. And he imagines that there is a kind of sympathetic relation between objects because they are like another. Having discovered in the lion the quality of courage or in the deer that of swiftness, he eats the flesh of the former that he may become bold, and of the latter that he may run well. So again, the image or picture or even the shadow of a man are imagined to be so closely related to him that it is possible by injuring them to injure the man himself. Even down to comparatively modern times we find traces of the belief that if the waxen image of a man be set in the heat to melt, the man himself will waste away.[1] Such illustrations might be multiplied. We have referred to them only as extreme cases of the part played by superficial analogies in the formation of illusory beliefs.

The English word "Analogy" comes from the Greek ἀναλογία, *analogia*; but the present meaning of the English word is different from that of its Greek original. Aristotle, for example, uses the word for what we call *proportion* in mathematics—an equivalence of ratios. In this sense it would be said that the relation of two to four is "analogous" to that of three to six. The principle of what we now call argument from Analogy is carefully analysed by Aristotle; but he calls it the argument from "example" (παράδειγμα, *paradeigma*). His view of its logical character is explained by Mellone, *Introductory Textbook of Logic*, Ch. VIII., § 4, pages 260-1. The question naturally arises: how did this change of meaning in the word "Analogy" come about? The change of meaning was a natural development. We may say that there is an "Analogy" between the Greek and the modern meanings of the word "Analogy" itself. In the purely mathematical sense we have an equivalence of ratios, $a : b :: c : d$. This means that there is a relation between c and d which is *essentially* (in this case *numerically*) the same as the relation between a and b, and if a, b, and c, are known numbers, and d unknown—so that we have $a : b :: c . x$, then we can infer x from c because of the identity of the relations; we infer $x = bc/a$. Now take the modern sense of the word. We have two cases S and P, not necessarily numerical or amenable to any kind of

[1] See D. G. Rossetti's poem, "Sister Helen."

mathematical treatment. Investigating S, we find it contains an important characteristic quality which can be analysed into a relation or connection of attributes MR. Investigating P, we find it also contains M and that M is also an important characteristic of it. Then by an analogical inference, we infer that P also contains R. Using the mathematical signs for an equivalence *of ratios*, not in the mathematical sense but as symbolising an equivalence *of relations* which may be non-mathematical, we have SM : SR :: PM : x; and, arguing from Analogy, we infer that x is PR. P therefore also contains the relation MR. The connection between the Greek and the modern meanings appears evident when we remember that the point in which S and P are similar is not a single unitary quality but can be analysed into a relation The examples given above (page 213) are effective illustrations of this view of the matter. For further discussion, see Joseph, *Introduction to Logic*, second edition, Ch. XXIV., pages 532-40.

5. Observation and Experiment

When we are in doubt as to the cause of any event, we make a supposition or conjecture—we imagine what seems to be a sufficient cause, and proceed to test it. Every research by which we seek to discover truth must be guided by some conjecture, whether it be a theoretical suggestion of a cause, or the practical suggestion of something to be accomplished.

We now come to the second of the two questions indicated above (page 209): methods by which a suggested cause may be tested.

(1) The first step in Science is to gain possession of the facts: this is impossible without "observation." Observation is a mental as well as a physical activity; for in order to observe, not only must the attention take a particular direction, but we must be more or less conscious of what we are looking for. In other words, observation, like ordinary perception, is **selective.** A man's experience consists, indeed, only of what he agrees to be interested in. Millions of events that pass before a man never enter into

his experience at all; they have no interest for him, and hence he does not notice them. It is a well-founded doctrine of modern psychology that without selective interest, experience would be an utter chaos. "Interest alone gives accent and emphasis, light and shade, background and foreground—intelligible perspective, in a word. Our own interest lays its weighty index-finger on particular items of experience, and may emphasise them so as to give to the least frequent associations far more power to shape our thoughts than the most frequent ever possess." And in science the interest springs from previous knowledge; the simplest fact, when noticed by a well-prepared mind, may become an observation of immense importance. The too-familiar anecdotes of James Watt's observation of the force of steam in lifting the kettle lid, and Newton's observation of the falling apple, will illustrate our point. The true *observer* brings to his observation more than he finds in it, and yet knows how to abandon one by one his most cherished preconceptions if the facts will not support them.

Observation is at bottom sense-perception. The possibilities of error in sense-perception arise from the fact that in perception things are not imaged in the mind as in a mirror—the mind itself contributes *meaning* to the result. There is no perception without an element of thinking, although in simple cases (for instance, the perception of a colour as red) we are scarcely conscious of it, and the whole sense-experience may seem to be immediately *given*. We need not dwell on this doctrine, which is well established in modern psychology. The more elaborate and systematic the observation is, the more extensive is the work of *thinking* in it. And it is in this thought-aspect of perception and observation that the sources of truth and error lie. Some writers describe this source of error as "a confusion of what we *perceive* and what we *infer* from what we

perceive." This suggests that the perception and the inference are two separate things, which is not the case. The confusion referred to is between the half-unconscious and instinctive inference, which experience has taught us to make correctly (*e.g.* "that is a man") and the more deliberate and conscious inference by which we extend the former (*e.g.* "that man is my friend Smith"). We often treat these secondary inferences as if they were as trustworthy as the primary ones, which is scarcely ever true.

An interesting illustration of what we have just said will be found in *The World of Life* by Alfred Russell Wallace (written in 1910, when the author's age was eighty-seven). He is speaking of "the most wonderful exhibition of migration-phenomena in the world—that of the small island of Heligoland." Most of the migratory birds from Scandinavia and Arctic Europe pass along the coasts of the North Sea; and the lighthouse on Heligoland serves as a guide, and the island itself as a resting-place. "The fact observed was that during the autumn migration, as regards many of the most abundant species, the young birds which had been hatched in the far North in the preceding June or July, and which therefore were only three or four months old, arrived in Heligoland earliest and alone, the parent birds appearing a week or two later. This is the fact. . . . The inference from this fact (apparently accepted by almost all European ornithologists) . . . is that these young birds start on their migration alone, and before their parents, and this not rarely or accidentally, but every year. They believe this to be a fact, and one of the most mysterious of the facts of migration . . . Yet the two things are totally distinct; and while I admit the fact observed, I totally reject the *inference* (assumed to be also a *fact*) as being absolutely without any direct evidence supporting it.

I do not think any English observer has stated that the young of our summer migrants all gather together in autumn and leave the country before the old birds; American observers state that *their* migrating birds do not do so; while many facts observed at Heligoland show that no such inference is required to explain the admitted facts . . . The fact of the young birds only appearing on Heligoland for the first week or so of the season of each species is easily explicable. Remembering that the autumnal migration includes most of the parent birds and such of their broods as have survived, it is probable that the latter will form at least half or, more often, two-thirds of each migrating flock. But the young birds, not having yet acquired the full strength of the adults, and having had little, if any, experience in long and continuous flights, a considerable proportion of them on the occasion of their first long flight over the sea, on seeing the lighthouse and knowing already that lights imply land and food-crops below them, and being also much fatigued, will simply drop down to rest just as they are described as doing. The old birds and the stronger young ones, however, pass high overhead, till they reach the north coast of Holland, or, in some cases, pass over to our eastern coasts. We must also remember that the longer the birds are in making the journey overland, the more young birds are lost by the attacks of birds of prey and other enemies. Hence the earliest flocks will have a larger proportion of young birds than the later ones. The earlier flocks also, being less pressed for time, will be able to choose fine weather for the crossing, and thus it will be only the young and quickly fatigued birds that will probably fly low and come down to rest. Later on, every recurrence of bad weather will drive down old and young alike for temporary shelter and rest. Thus, all the facts are explained without having recourse to the

wildly improbable hypothesis of flocks of immature birds migrating over land and sea quite alone and a week in advance of their parents or guides."

Whether Dr. Wallace's explanation of the appearance of the young birds by themselves is accepted or not, this discussion is an effective example of the importance of distinguishing the interpretative inference from the observed fact.

(2) It is equally important to distinguish observation and **experiment.** In pure observation, facts observed are due to Nature; in experiment, they are to a great extent arranged by ourselves. Jevons has excellently described the difference between the two. "To *observe* is merely to notice events and changes which have been produced in the ordinary course of Nature, without being able or at least attempting to control those changes. Thus the early astronomers observed the motions of the sun, moon, and planets among the fixed stars, and gradually detected many of the laws or periodic actions of those bodies. Thus it is that the meteorologist observes the ever-changing weather, and notes the height of the barometer, the temperature and moistness of the air, the direction and force of the wind, the height and character of the clouds, without in the least being able to govern any of these facts. The geologist, again, is generally a simple observer when he investigates the nature and position of rocks. The zoologist, the botanist, and the mineralogist usually observes animals, plants, and minerals as they are met with in their natural condition. In *experiment*, on the contrary, we vary at our will the combinations of things and circumstances, and then observe the result. It is thus that the chemist discovers the composition of water by using an electric current to separate its two constituents, oxygen and

hydrogen. The mineralogist may employ experiment when he melts two or three substances together to ascertain how a particular mineral may have been produced. Even the botanist and zoologist are not confined to mere observation; for by removing animals or plants to different climates and different soils, and by what is called domestication, they may try how far the natural forms and species are capable of alteration."

We must remember that it is impossible to draw a line of complete separation between the two processes, so as to say "just here the one ends and the other begins." They have this much *in common*: observation (as we have pointed out) is itself not only an active but a selective process, and in this it resembles experiment. If we look for a transition-process leading from observation to experiment, and sharing the characteristics of both, we can find it in the use made of **instruments of observation** such as the telescope and the microscope. Such instruments immensely extend the range of our sense-perception, as in the case of objects very distant or very minute; and the photographic camera comes to the aid of the astronomer, not only by detecting facts which would be inaccessible to the human eye using the same telescope, but also by providing a permanent record of them.

Still, experiment, in the full sense of the word, goes much further and deliberately arranges the facts in order to obtain knowledge regarding them. A scientific experiment is a "question asked of Nature." It is *the construction of a typical and crucial case, on a plan thought out in advance, in order to test a hypothesis or decide a question which has arisen.*

The advantages of experiment are very great. Jevons has arranged them under three heads.

(*a*) "We might have to wait years or centuries in order to meet accidentally with facts which we can readily produce at any moment in a laboratory; and it is probable that most of the chemical substances now known, and many excessively useful products, would never have been discovered at all by waiting till nature presented them spontaneously to our observation." Here Jevons refers to the infrequency or rarity of certain facts of Nature, even very important ones.

He then goes on to speak of the minuteness of many phenomena which makes them escape ordinary experience.

(*b*) "Electricity operates in every particle of matter, at every moment of time; and even the ancients could not but notice its action in the 'loadstone,' in lightning, in the Aurora Borealis, or in a piece of rubbed amber. But in lightning electricity was too intense and dangerous; in the other cases it was too feeble to be properly understood. The science of electricity and magnetism could only advance by getting regular supplies of electricity from the common 'electric machine' or the galvanic battery and by making powerful electromagnets."

Jevons then deals with the importance of studying, under varied conditions, phenomena which Nature presents to our observation in the same general form:—

(*c*) "Thus carbonic acid is only met in the form of a gas, proceeding from the combustion of carbon; but when exposed to extreme pressure and cold, it is condensed into a liquid, and may even be converted into a snowlike solid substance. Many other gases have in like manner been liquefied or solidified, and there is reason to believe that every substance is capable of taking all three forms of solid, liquid, and gas, if only the conditions of temperature and

pressure can be sufficiently varied. Mere observation of nature would have led us, on the contrary, to suppose that nearly all substances were fixed in one condition only, and could not be converted from solid into liquid and from liquid into gas."

The advantages of experiment, as set forth in the preceding passage, may be thus summed up:—

(*a*) It saves the investigator from any long waiting on Nature; experiments can be repeated as often as is necessary.

(*b*) It enables the investigator to study (under conditions arranged by himself) processes which, as they occur in Nature, are too swift and transient, or too minute, to be dealt with on the basis of mere observation.

(*c*) It enables the investigator to study a phenomenon under varied conditions in order to ascertain what circumstances are essential, and what are not essential, to the production of the various changes of which it is capable.

We can distinguish the sciences according to the extent to which they depend upon experiment in this sense. Without experiment, Mechanics, Chemistry, and Physics could scarcely exist; and these are fundamental sciences in an advanced state. In Physiology and in Psychology experiment naturally plays a smaller part; but in Psychology the possibilities of experimental work have been actively exploited, in recent years, in every branch of the subject. In the descriptive and classificatory sciences—Mineralogy, Botany, Zoology—the possibilities of experiment are least. Astronomy is in a peculiar position. Obviously *we* cannot experiment with the heavenly bodies; but Nature herself, from time to time, produces unusual phenomena from which knowledge, valuable theoretically

and practically, may be arranged—as in the case of an eclipse of the sun. Such events have been called "natural experiments."

6. Methods of Investigation

When a possible cause has been suggested, conjecturally or otherwise, we can distinguish and classify the scientific methods by which the hypothesis may be tested. We shall see that there is a principle common to all these methods, namely, to vary as much as possible the circumstances of the event under investigation, so as to eliminate what is unessential or casual in them.

We shall explain only the simpler methods by which hypotheses of immediate causation may be tested or investigated. They may be classified as follows:—

(1) The Method of Single Agreement.

(2) The Method of Double Agreement.

(3) The Method of Single Difference.

(4) The Joint Method of Difference and Agreement.

(5) The Method of Concomitant Variations.

(6) The Method of Residues.

(1) When our object is to discover the cause *of a given effect*, and our *control* over the circumstances under observation is very limited, so that *experiment*, unless of an extremely rudimentary kind, is not possible, then the method to be adopted may be formulated in the following rule: *When observation shows that two events accompany one another (either simultaneously or in succession), it is probable that they are causally connected; and the probability increases with the number and variety of the instances.* The reader should notice the difference between this method and the

method of enumeration, based on the *counting* of instances (see page 209). In this method stress is laid on the variety as well as on the number of the instances; to increase the probability, we must deliberately assemble not only as many but as varied instances as possible.

This may be called the **Method of Single Agreement.** Some writers represent it symbolically by using different letters to represent different circumstances in which the event occurs. Thus, suppose we are able to analyse the various instances of A as follows: first instance, A*b*C*d*; second, AC*fe*; third, A*gh*C; and so on: then C is the only other fact in which the instances of A agree; hence there is a probability that A and C are causally connected. On this, an important critical remark is necessary. To denote the facts learnt through observation by letters, in this way, is to take for granted that the hardest part of the work of observation is already done. When events have been analysed into their elements in this manner, it is a very simple affair to ascertain the common facts in the different instances. Nature not only fails to show us, at a glance, what events are really connected with a given one; she does not give us events marked off into distinct and separate facts. And for this reason also it is very difficult to be sure that the instances have *only one* material circumstance in common.

The "plurality of causes" is also a serious obstacle to the Method of Single Agreement. We have seen (page 202) that in most cases the plurality disappears before a more searching analysis; but still, there is a sense in which it is true—for instance, that heat, or light, or motion, may be caused in various ways. If heat is produced by friction, and by combustion, and by electricity, all these real causes would be eliminated by this Method, for they are points in which the different instances of heat differ.

Practically, as we have seen, the force of the Method depends on the number and variety of the instances; the more numerous and varied they are, the greater the probability that A and C are causally connected.

We add two comparatively simple examples of the use of this Method. The subject of the first example is the injuriousness of the north-east wind in this country. Occasional well-known causes of a wind being injurious are its violence; its excessive heat or cold; its excessive dryness or moisture; its electrical condition; its being laden with dust or exhalations. Let the *hypothesis* be that the last is the cause of the north-east wind's unwholesome quality, for we know that it is a ground-current setting from the pole towards the equator and bent westward by the rotation of the earth; so that, reaching us over thousands of miles of land, it may well be laden with dust, effluvia, and microbes. Now when we examine many cases of north-east wind, we find that this is the only circumstance in which they all agree; for it is generally dry, but sometimes wet; sometimes cold, sometimes hot; sometimes light, sometimes violent; and of all electrical conditions. Each of the other circumstances, then, can be omitted without the north-east wind ceasing to be injurious; but one circumstance is never absent—namely that it is a ground current. This therefore is *probably* the cause of its harmful quality.[1]

Among other things, this illustrates the purely *observational* character of the Method of Single Agreement. The following example illustrates the importance of increasing and varying the number of instances. A person might suppose that the peculiar colours of mother-of-pearl were due to the chemical qualities of the substance.

[1] Alexander Bain, *Inductive Logic*, Bk. III., Ch. vi.

Much trouble might have been spent in following out that notion by comparing the chemical qualities of various iridescent substances. But Brewster (about 1845) accidentally took an impression of mother-of-pearl in a cement of resin and bees-wax, and finding the colours repeated upon the surface of the wax, he proceeded to take other impressions in substances the chemical composition of which was in every case different; and he found that the iridescent colours were in every instance the same. He concluded that the composition of the substance was a matter of indifference, and that the form of the surface is the real cause of the colours. The surface, examined with a microscope, presented a grooved structure (the grooves being sections of all the concentric *strata* of the shell): and these minute grooves, according to the known laws of reflection and refraction of light, produce the colours. The conclusion in this example is very reliable because it was possible actually to manufacture the variety of instances required.

(2) We must now notice an important modification of the Method of Single Agreement, which strengthens it, but does not deprive it of its purely observational character. When by this Method we have established the probability that the suspected agent A is causally connected with the event C, by the number and variety of the cases in which they occur together, we may then examine as many cases as possible, in the same field of inquiry, where the suspected agent is absent; and if these agree also in the absence of the event C, we have a double application of the Method of Agreement, which may therefore be called the **Method of Double Agreement**; and the suggestion made by the first application of the Method, that A and C are causally connected, is very considerably strengthened. The cases

in which A and C are present together are called the **positive instances**; and the cases in which A and C are absent together are called the **negative instances.** It is obvious that the positive and negative instances must be *in pari materia,* in other words, as we said above, in the same field of inquiry: *e.g.* if the subject is one belonging to Geology, then the negative as well as the positive instances must be sought in the department of Geology.

The Method of Double Agreement is summarised in the following rule: *Whatever is present in numerous observed instances of the presence of an event, and absent in numerous observed instances of its absence, is probably causally connected with that event.* The Method assumes that we have a wide and varied experience of the conjunction of the two events, and that we have failed to find any instance where one has occurred without the other. Then it is probable that they are causally connected, and the probability increases with the number and variety of the instances. It presupposes the ordinary Method of Single Agreement before proceeding to " marshal the negative instances "; and like that Method, it is appropriate where exact experiment is not possible. The Method of Single Agreement leads us to suspect a certain cause for a given event; we then look for instances, in the same field, all agreeing in the absence *both* of the effect and of its suspected cause.

In the example of the north-east wind, given above, the negative instances would be looked for by examination of other winds, for instance the south-west wind, which does *not* blow for long distances comparatively near the surface of the ground, and which is *not* injurious. In the example of the iridescent colours produced by the minute grooves on the surface of mother-of-pearl, where the grooves are surfaces of all the concentric *strata* of the shell, it is found that when the actual surface of any one *stratum* is examined

(and the surface of the pearl itself is such a *stratum*) the colours are not seen.

One of the most remarkable examples on record of the Method of Double Agreement is found in Darwin's investigation of the theory that " vegetable mould " is produced by earth-worms. He devoted a special treatise (*Formation of Vegetable Mould through the Action of Earthworms*, 1881) to the proof that these creatures are thus performing a work of vast magnitude and importance for the maintenance of life on the surface of the earth. Essentially the investigation led to the establishment of two propositions: (*a*) many plots of land, all of them containing multitudes of earth-worms, though otherwise very different in character, became covered increasingly with vegetable mould; (*b*) many plots of land not unlike the former plots as a whole, but deficient in earth-worms, were not covered with vegetable mould. The most important steps in the investigation are described in Mellone, *Introductory Textbook of Logic*, Ch. IX., § 6.

(3) When a causal connection has been *suggested* by the Method of Single Agreement, or by the previous knowledge and experience of the investigator, or in some other way, an important means of testing the supposition is provided in the **Method of Single Difference.** Experiment is characteristic of this Method. When we can produce the effect ourselves, we are not content with the more general probability which the Method of Agreement yields. We take the agent believed to be the cause, and introduce it into definite circumstances arranged by ourselves; and we conclude that whatever change follows must be due to the agent which we have introduced. Sometimes we add the agent to the known circumstances, at other times we subtract it; logically the results are the same. The rule of the Method of Single Difference may be thus stated: *When the addition of an agent is followed by the appearance, or its subtraction by the disappearance, of a certain event, other circumstances remaining the same, that agent is causally connected with the event.* When the suspected agent is

present, we have the positive instance; when it is absent the negative instance.

The Method of Difference plays a great part in our everyday inferences. We use it unconsciously whenever we introduce some definite agency or change, which is followed by a definite consequence. We drink water, and there is a cessation of thirst. We make a noise, and a sleeping child wakes. We rub a match, and it bursts into flame.

The records of every experimental science, and above all those of Chemistry, provide abundant examples of the Method of Difference. One of the simplest scientific examples is seen in the coin and feather experiment, designed to show that the resistance of the air is the cause of a light article, as a feather, falling to the ground more slowly than a heavier one, as a coin. The event to be investigated is the retardation of the feather. When the two are dropped simultaneously in the receiver of an air-pump, the air being left in, the feather flutters to the ground after the coin. This is the positive instance, where the suspected cause is present, and the effect occurs. Then the air is pumped out of the receiver, and the coin and feather, being dropped at the same instant, reach the base together. This is the negative instance, where the suspected cause is absent, and the effect does not occur. The single circumstance of difference is the presence of the air in the former case, and with its removal the retardation of the feather's fall is removed.

It has been truly observed that the method of Single Difference assumes various forms according to circumstances. Sometimes—as in the example which we have just given—the two instances compared are really two successive conditions of the same set of circumstances, to which something is added to obtain the positive instance,

or from which something is withdrawn to obtain the negative instance. Sometimes the two instances are separate cases which resemble each other in all material circumstances except one, as indicated in the usual symbolisation:—

$$\text{Positive instance} \begin{cases} a \ b \ c \ d \\ m \ n \ p \ r \end{cases}$$

$$\text{Negative instance} \begin{cases} b \ c \ d \\ n \ p \ r \end{cases}$$

from which the causal relation *a–m* is inferred. Sometimes, again, the instances compared are really groups of instances, each group being treated as a single instance. "When Pasteur tested the efficacy of preventive inoculation against anthrax, which he thought he had discovered, he first inoculated twenty-five sheep with a mild preparation of the 'serum'; and then he inoculated them again, and also twenty-five other sheep, which had not been inoculated before, with a strong preparation of the 'serum.' The twenty-five sheep which had undergone the preparatory inoculation all survived, while the others all perished. The two 'instances,' in this case, were not single sheep, but two groups of twenty-five sheep in each; and the result was all the more conclusive."[1]

The reader will see, on consideration, that the successful application of the Method of Differences depends on our knowledge and control of the negative instance. If we have insufficient *knowledge* of the material circumstances acting in the negative instance, we cannot be sure that when the suspected cause is introduced, its apparent effect is not due to some condition already present and unknown to us; and if we have insufficient *control* of the negative

[1] See Wolf, *Textbook of Logic*, Ch. XIX., § 3, p. 209.

instance, we cannot be sure that the introduction of the suspected cause has not so far disturbed the other conditions that part of the observed effect is due to them. For example, if the attempt were made to measure the force of gravity by delicately suspending a small and light ball, and then bringing a large and heavy ball close to it, the mass of the large ball would attract the small one. But the experiment would not be of the least value unless performed with the utmost precaution; the sudden motion of the large ball would cause currents of air, vibrations, etc., which would disturb the small ball far more than the force of gravity.

The Method of Single Difference proves *a* cause; but to prove that this cause is the only cause, we require to pay special attention to the possible negative instances.

(4) The position which we have now arrived at is this. In the case of a suspected causal connection between A and C, we require to establish the two propositions:—

If A, then C;
If not A, then not C.

And in order to prove the second of these two propositions, it is usually necessary to conduct an independent investigation into as many of the relevant or material negative instances as is possible; and by "relevant or material" negative instances we mean those which fall within the same field of inquiry. This "independent investigation" of the negative instances always presupposes the employment of the Method of Single Difference, and supplements it in the way in which we have indicated. For the reasons explained above, it is usually most important and most necessary that the Method of Single Difference *should be* thus supplemented; and when this is done, we have what is practically a new Method, which may be called **the Joint**

Method of Difference and Agreement. The " Difference " refers to the causal connection experimentally determined in the positive instance; the " Agreement " refers to the *absence of the suspected cause and of its effect* in *all* the negative instances examined. This is the fundamental method for every scientific inquiry into Causation. The difference between the Method of Double Agreement and the Joint method of Difference and Agreement, consists in the purely observational character of the former, and the essentially experimental character of the latter. In the latter Method, the negative instances have often to be *constructed* in such a way that the cause cannot occur in any of them. Its rule may be thus expressed: *When one event has been shown to be the cause of another by the Method of Single Difference, and when no instance in which the one occurs without the other can be found or constructed : then it is probable that the first is the only possible cause of the second; and the probability increases with the number and variety of the negative instances.* The extent of the field over which we must range in assembling negative instances is a question which the trained investigator, possessing wide and accurate knowledge of the subject, alone can decide. In Chemistry, there is reason to believe that we have experimental knowledge of nearly all the elements to be found on earth. Hence when by the Method of Single Difference an element yields a particular reaction (" If A, then C "), the investigator is justified in assuming that our knowledge of the negative instances (the properties of the other elements) warrants the conclusion that no different element will produce the same reaction (" If not A, then not C "). But the limited number of the elements places Chemistry in an exceptional position. In other branches of science, the great difficulty lies in the examination of the negative instances.

The following examples show how sometimes the whole question turns on the construction of adequate negative instances. The question at issue was the theory of "spontaneous generation," *i.e.* the production of living from non-living matter, as inferred from the appearance of life in some infusions. It was believed by some observers that "infusoria" (microscopically minute forms of life) arose spontaneously in infusions of decaying animal or vegetable matter. Needham—an English priest in the eighteenth century—claimed to have obtained infusoria in jars which had been filled with putrescible liquids, boiled, and then closed. But he had closed the jars with cork stoppers sufficiently porous to let micro-organisms through from the air. A contemporary opponent repeated the experiment with greater care. He used hermetically sealed jars, and kept them for an hour in boiling water. He then opened them and examined their contents with the microscope, and found not the slightest trace of infusoria.

For other illustrative examples of investigations akin to the one which we have just described, see Mellone, *Introductory Textbook of Logic*, Ch. IX., § 3, page 298*b* (Method of Single Agreement), and § 7, pages 310-11 (Joint Method); and for a most instructive analysis of an elaborate experimental investigation of the theory of "spontaneous generation" by Pasteur, see Stebbing, *A Modern Introduction to Logic*, Ch. XVII., § 2. The nature of the investigation is not too abstruse for the "lay" reader to follow.

(5) In the most exact sciences, where the causes and effects which we examine are susceptible of degrees of intensity, or at any rate of being "more or less," we may not only observe and compare events but *measure* them. In fact, the generalisation stated by Jevons is fully justified: "Every question in physical science is at first a matter of fact only, then a matter of quantity, and by degrees becomes more and more precisely quantitive." As soon as

events can be measured, there arises the possibility of a quantitative form of either of the two primary Methods (Single Agreement and Single Difference). This is known as **The Method of Concomitant Variations.** Its rule may be thus stated: *Any event which varies quantitatively whenever another event varies quantitatively is causally connected with it.* This may be regarded as a modification of the Method of Single Agreement or of the Method of Single Difference, according as the variations are ascertained by experiment or not. In the case of Single Difference, the other relevant circumstances of the event are assumed to be the same, as is required by the logical nature of this Method. In the case of Single Agreement, the other relevant circumstances vary in the various instances; and we must add another clause to the rule: *other circumstances varying independently or with no correspondence.*

The method may be applied where *exact measurement is not possible.* It is available whenever the intensities of two events can be compared as they vary from more to less or the reverse. Moreover an important case for its application is when an event goes through *periodic changes, i.e.* alternately increases and decreases; of this, the tides are a conspicuous example, in their relation to the apparent motions of the sun and moon.

An elementary illustration of Concomitant Variations is seen in the familiar thermometer. Every perceptible increase in the temperature of the surrounding atmosphere is accompanied by a perceptible increase of the volume of mercury in the tube, and *vice versa;* and we reach the conclusion that the volume of mercury is causally connected with the temperature of the surrounding medium. If we had *only this particular series* of observations on which to base a conclusion, we could not say more than that the

respective variations are " causally connected "; we could not say, on these facts alone, which is " cause " and which is " effect." But the general laws of the expansion and contraction of bodies with variations of temperature are already known, and that mercury remains fluid within the limits of normal temperatures on the surface of the earth.

This example also reminds us that Concomitant Variations is a Method peculiarly appropriate when neither of the events under investigation can be altogether eliminated. Take the case of the *friction of moving bodies.* No case is known of frictionless motion; but the amount of friction can be very much reduced, and gradually reduced, under experimental conditions. It is then found that the less the amount of friction which a moving body encounters, the longer is the time during which its motion continues; the period of motion and the amount of friction " vary inversely." The conclusion is that in what is technically termed " the limiting case," that is, in this example, if friction could be eliminated altogether and no other obstacle intervened, a body once in motion could continue to move indefinitely.

The application of this to the proof of what is called the " First Law of Motion " has been thus explained by John Stuart Mill. The law states that all bodies in motion continue to move in a straight line with uniform velocity until acted on by some other force. " This assertion," said Mill, " is in open opposition to first appearances; all terrestrial objects, when in motion, gradually abate their velocity and at last stop . . . Every moving body, however, encounters various obstacles, as friction, the resistance of the atmosphere, etc., which we know by daily experience to be causes capable of destroying motion. It was suggested that the whole of the retardation might be owing to these causes. How was this inquired into? If the obstacles

could have been entirely removed, the case would have been amenable to the Method of Difference. They could not be removed, they could only be diminished, and the case, therefore, admitted only of the Method of Concomitant Variations. This accordingly being employed, it was found that every diminution of the obstacles diminished the retardation of the motion; and inasmuch as in this case the total quantities both of the antecedent and consequent were known, it was practicable to estimate, with an approach to accuracy, both the amount of the retardation and the amount of the retarding causes of resistance, and to judge how near they both were to being exhausted; and it appeared that the effect dwindled as rapidly as the cause, and at each step was as far on the road towards annihilation as the cause was. The simple oscillation of a weight suspended from a fixed point, and moved a little out of the perpendicular, which in ordinary circumstances lasts but a few minutes, was prolonged in certain experiments to more than thirty hours, by diminishing as much as possible the friction at the point of suspension, and by making the body oscillate in a space exhausted as nearly as possible of its air. There could therefore be no hesitation in assigning the whole of the retardation of motion to the influence of the obstacles; and since, after subducting this retardation from the total phenomenon, the remainder was a uniform velocity, the result was the proposition known as the First Law of Motion."

When the Method of Concomitant Variations is used, not only to establish the fact that there is some causal connection between two events, but to discover the law according to which the phenomena vary, great caution is required as soon as the generality of the conclusion goes beyond the limits of the variations actually observed. For example,

water as it approaches near to " freezing point," instead of continuing to contract as it becomes colder, begins to expand. The caution which this particular fact suggests is one of very wide range. When our observations by this Method have been taken within comparatively narrow limits, there must be an element of uncertainty. That is why the application of the Method of Concomitant Variations to social inquiries—especially to the interpretation of what are called " statistical correlations "—requires such care and caution.[1]

(6) We now come to the last of the six Methods named above. Since the time of John Stuart Mill it has been known as **The Method of Residues.** But the account of it given by Mill and his followers involved a confusion of two entirely different questions.

(*a*) So far as " Residues " can be called a " method " at all, it is obviously deductive, not inductive. Its rule is as follows: *Subtract from any complex event such part as is known by previous inductions to be the effect of certain antecedent events; then the residue of the event is the effect of the remaining antecedents.* We say " subtract " because the method is essentially quantitative, involving measurement; and the only satisfactory illustrations of it are of that kind. Thus if we know that the complex event *efgh* is caused by ABCD, and is *caused only* by ABCD; and that *e* is caused by A, and only by A; *f* by B; and *g* by C: then we know that *h* is caused by D. The reader who is acquainted even with the elements of any of the exact sciences—especially Chemistry—will experience no difficulty in finding illustrations of this rule. An almost too familiar example arises

[1] See Wolf, *Textbook of Logic*, Ch. XX., § 2 (*cp.* Ch. XIX., § 4); Stebbing, *Modern Introduction to Logic*, Ch. XVIII., § 3.

out of the estimation of weights. Thus, suppose it is required to estimate the weight of a certain volume of liquid. The vessel which is to contain it is carefully weighed when dry and empty, and then weighed again when containing the liquid in question. The weight of the liquid is of course this total weight *less* the weight of the vessel.

(*b*) The term " Method of Residues " has also been used to designate a cautionary rule which is not a " method " at all, for it prescribes no *method* of inquiry, but defines a kind of case where further inquiry is urgently needed. The rule may be thus stated: *When any part of a complex event is still unexplained by the causes which have been assigned, a further cause for this remainder must be sought.* There is no indication in the inquiry, so far as it has gone, of *what* this cause actually is, though there may be some indication of its general nature. The " rule " is a finger-post to the unexplained. It calls attention to " residual phenomena " which have to be accounted for. Such phenomena have frequently led to discoveries of the first importance.

Such was the discovery of the planet Neptune. From 1804 it had been noticed that the orbit of the planet Uranus—at that time supposed to be the outermost of the planets—was subject to an amount of disturbance which could not be accounted for from the gravitational influence of the known planets—Jupiter and Saturn being the only two of these exercising any perceptible effect on Uranus. Astronomers therefore suspected the existence of an unknown planet, and they were faced with the difficult " inverse problem," given the disturbances, to find the orbit and position of the disturbing planet. The problem was successfully solved by two different astronomers

(J. C. Adams in Cambridge and U. J. Leverrier in Paris) working quite independently. They succeeded in assigning a direction to this body, close to which " Neptune " was actually found in September 1846. When the gravitational effect of this new planet was included, the anomalies in the orbit of Uranus were almost removed. But they were not quite removed; there were still some " residual " irregularities. A certain systematic character in them convinced the American astronomer Lowell that they implied the gravitational action of a further unknown body, and by unsparing labour he assigned a position and magnitude to it. He felt as much confidence in his work that he provided after his death for the prolonged search that was necessary. Twelve years later, in 1930, a body was found agreeing strikingly with his forecast. He was wrong only about its magnitude, and this delayed success in the search. All these vast systematic calculations of the " gravitational " actions of the bodies in question, were based upon Newton's Law of Gravitation—that any two bodies attract each other with a " force " varying (*a*) inversely as the square of the distance between them, and (*b*) directly as the product of their masses.

The deductive " Method of Residues " is valuable in cases where the effects of different causes (acting together) unite in a homogeneous total—in other words, are all blended together, producing a joint effect of the same kind as the separate effects. This is called the " intermixture of effects." We want to know how much of the joint effect is due to each of the several causes. And when the magnitude of the several known causes can be quantitatively estimated, and the magnitude of their respective effects deduced, the Method of Residues is available.

The " intermixture " or combination of homogeneous effects may assume very complicated forms. The action of

any cause may be (*a*) augmented, (*b*) diminished, (*c*) diverted, (*d*) wholly counteracted, by that of another cause. Thus, for simple examples we may suppose a body (*a*) pulled by two forces in exactly the same direction; (*b*) pulled by two forces of different magnitudes in exactly opposite directions; (*c*) pulled in one direction by one force, and by another force pulled in a direction at right angles with the former; (*d*) pulled by two *equal* forces in exactly opposite directions, when no motion takes place. In every case of the combination of effects, *each cause produces its appropriate effect*, even though the effect may disappear wholly or partly in the total result. An object may remain at rest, when subject to two equal forces acting in opposite directions, but we cannot say of either of these forces that it is inoperative. Each prevents any visible effect resulting from the other; but then this is the very effect which it produces. When the full consequences of a Law of Nature are thus affected (modified or neutralised) by other Laws, it is called a *tendency*. There are no real *exceptions* to Laws of Nature. A Law of Nature cannot be violated; but, as we have seen, its consequences can be modified or neutralised.

If, at a later stage, the reader studies John Stuart Mill's exposition of "Induction" in the third Book of his *System of Logic*, he will find that Mill formulated five "canons" or rules for inductive investigation, which he called (1) the Method of Agreement; (2) the Method of Difference; (3) the Joint Method of Agreement and Difference; (4) the Method of Concomitant Variations; (5) the Method of Residues. For these methods Mill made high claims, which in other parts of his work he was obliged to retract; and his technical formulation of the "canons" is open to much criticism. The names which Mill gave to these methods are still in general use; and for

the reader's guidance we indicate their relation to the rules for inductive investigation which we have explained above. Mill's "Method of Agreement" and "Method of Difference" correspond in principle respectively to what we have called the "Method of Single Agreement" and the "Method of Single Difference." Mill's third method, which he calls the "Joint Method of Agreement and Difference" is not a "joint" method at all and not a combination of "Agreement" and "Difference"; it is simply a double application of the "Method of Agreement," and it corresponds in principle to what we have called the "Method of Double Agreement." Mill's "Method of Concomitant Variations" and "Method of Residues" correspond in principle to the methods which we have called by the same names.

In many of the textbooks which are in current use, we find that the fundamental questions of Inductive Logic are far too much intermingled with exposition and criticism of Mill's *Logic*. The study of Mill's classical statement of the Logic of Empiricism[1] is valuable for the student who has already grasped the fundamental principles of Induction; but the study of Mill is not a suitable mode of introduction to the subject. It is a sign of strength in Professor A. Wolf's *Textbook of Logic*, that he carries through a lucid exposition of the essentials of scientific method without any preoccupation with Mill's arguments. A careful treatment of the main principles of Inductive Logic will be found in E. M. Whetnall's edition of Welton and Monahan's *Intermediate Logic*, Ch. XXIV-XXXIII., and in Stebbing, *A Modern Introduction to Logic*, Ch. XIII. to Ch. XXI. (see also Ch. XXV., § 3). The advanced student will find W. E. Johnson's *Logic* of great value (see especially Vol. II, Ch. VIII. to XI., and Vol. III.). Some very inaccurate statements are still current about Aristotle's theory of Induction. What Aristotle called the "Inductive Syllogism" is not a contribution to Inductive Logic as we understand it (Mellone, *Introductory Textbook of Logic*, pages 246-7; and Joseph, *Introduction to Logic*, second edition, pages 378-80). Aristotle's contributions to Inductive Logic are to be found in parts of his logical treatise called the *Topics* (Joseph,

[1] By "Empiricism," in this connection, is meant the doctrine that all knowledge at bottom *consists* of "experience," and that all "experience" at bottom *consists of* the data of sense-perception.

op. cit., pages 386-90), and in his doctrines of the Enthymeme and the Paradeigma (Mellone, *op. cit.*, pages 252-61).

(7) We must now ask, how far do the methods, which we have already explained, carry us? The answer is, they cannot do more than establish a causal law that a consequent C results from an antecedent A under all circumstances, *i.e.* that A is the only possible cause of C; and this can be done only by the most powerful of the methods, that of Joint Difference and Agreement, and then only when we have control of all the relevant negative instances. What more than this do we want? We want, if possible, to prove the connection between this Law and other scientific laws, and more especially to deduce it or anticipate it on the ground of previous knowledge.

When a Law is ascertained, and we do not know how to connect it with other laws, it is called an **empirical Law.** Mill said that "scientific inquirers give the name of 'empirical laws' to uniformities which observation or experiment has shown to exist, but on which they hesitate to rely in cases varying much from those which have been actually observed, for want of seeing any reason *why* such a law should exist." When we speak of the "reason why" a fact or law exists, we mean by its "reason" its connection with other facts or laws. We may therefore say that the laws of causation suggested or established by the Method of Direct Induction cannot be more than "empirical laws"; and the degree of reliability varies with the power of the Method by which it was established. Many empirical laws are scarcely more than cases of *enumeration* in the sense already defined (see page 209). The following are examples of empirical laws of which no explanation—no "reason *why*," as Mill expressed it—is as yet forthcoming: the apparent exceptions to the almost universal truth that bodies expand with increase of temperature;

the law that gases have a strong tendency to permeate animal membranes; the law that when different metals are fused together the alloy is harder than any of the components; the law that substances containing a very high proportion of nitrogen (such as morphia and hydrocyanic acid) are powerful poisons; the *local* laws of the flow and ebb of the tides in different places. (The connection between changes in the height of the tides and the changing positions of the sun and moon with reference to each other and the earth, would be a mere empirical law if it were not deducible from the gravitational attraction of the sun and moon upon the waters of the earth.)

Descriptive statements of this kind abound in "Natural History"; thus, any accurate description of the process of *growth* is only an exposition of empirical uniformities, mere laws of co-existence and sequence. The following very summary statement may serve as an illustration. A "dicotyledonous" seed, such as the common garden bean, when sown in suitable soil under favourable conditions of moisture, temperature, and light, will as a rule germinate. The embryo which it contains will protrude its rootlet; this will grow downwards, and from it will spring the primary root which fixes the developing seedling in the soil. This root may branch repeatedly and ultimately form a complex root-system. The two cotyledons will (in many cases) emerge from the seed-coat and develop into green leaves. A stem will grow upward. When an embryonic "plumule" is present (usually where the cotyledons are linked together) this will develop its rudimentary leaves into the first foliage-leaves of the growing plant. The stem, like the root, may branch repeatedly, and will give rise to more foliage-leaves, and ultimately to the reproductive shoots known as flowers. The latter will (in many cases) be bisexual, containing both ovules and

pollen. If the pistil is pollinated, this will normally give rise to further processes which will lead to the maturing of the seeds. Each seed will contain an embryo capable of developing into a form similar to the parent plant.

In connection with the subject of "empirical laws," reference should be made to our discussion of "uniformities of co-existence" (page 208).

7. Indirect Induction

The Methods of Direct Induction play a large part in the early stages of almost every science, and continue to do so, above all, in the biological and social sciences, and (we may say) in every branch of science in which our generalisations, or statements of causal law, are not far removed from the observed facts. By "not far removed from" we mean not highly abstract as compared with the concrete facts of observation. But physical science—and especially the science known as "Physics," which investigates the constitution of "matter"—has become more and more abstract in the sense which we have just indicated; and the course of its progress has revealed with increasing clearness the importance of a more elaborate scientific method, in which Induction and Deduction are combined. We have now to see what are the essential steps in this method.

The essential steps in the process of Indirect Induction are these:—

(*a*) Preliminary ascertainment of facts or causal laws by observation or experiment.

(*b*) Formation of a hypothesis as to the nature of the connection embodied in the facts.

(*c*) Deductive reasoning (including mathematical calculation) on the basis of the hypothesis, which is treated as a general principle from which conclusions are drawn.

(*d*) Verification, or comparison of these consequences with the facts.

The investigator has before him the "facts"—the results of observation and experiment. He comes to them with a prepared mind. But the case cannot be met by a simple hypothesis of causation which can be directly tested by the methods already described. Indeed, these methods themselves may have actually led to the formulation of the problem which he is now studying. His prepared mind forms a hypothesis as to the nature of the connection embodied in the facts. He then draws out the consequences of the hypothesis by deductive reasoning. If, as is usually the case, the assumed connection is a quantitative one, *i.e.* is a matter of measurement, he deduces its consequences by mathematical calculation, until he arrives at such conclusions as can be put to the test of observation or experiment.

The following is a comparatively simple example. Galileo,[1] during the period in which he held the post of mathematical lecturer in the University of Pisa, carried out a remarkable series of experiments by which he established the first principles of dynamics. By dropping metal balls of different weights from the leaning tower of Pisa, he disproved the current assumption that the velocity of falling bodies varied with their weight (*i.e.* that heavier bodies fell more quickly than lighter ones, allowance being made for the resistance of the air). He afforded his opponents an ocular demonstration of the falsity of this

[1] Galileo was born in Pisa, February 1564, and died in Florence, January 1642.

assumption. He then set himself to ascertain the real law of the velocity of falling bodies. At length he arrived at the hypothesis that a body falling from rest would fall with uniform acceleration (*i.e.* at a uniformly increasing rate), and that the distance through which it fell would be proportional to what is called "the square of the time" occupied by the fall—in other words, the number of seconds multiplied by itself. He could not test this hypothesis by direct experiments with freely falling bodies, because the rate of fall would be too fast. But he could test it by rolling balls down grooves on an inclined plane, and by this means he arrived at an experimental verification of his hypothesis.

In one second a body falling freely near the surface of the earth, and not appreciably resisted by the air, is found to drop 16 ft. In two seconds it drops 64 ft.; in three seconds it drops 144 ft.; in four seconds, 256 ft; in five seconds, 400 ft.; and so on. That is, the distances fallen through in 1, 2, 3, 4, 5, etc., seconds are proportional to the numbers 1, 4, 9, 16, 25, etc. All this was experimentally established by Galileo.

We now proceed to deal with the question, What are the characteristics of a good hypothesis? The answer to this question will afford a fuller understanding of what we have called the "essential steps" in the process of Indirect Induction.

(1) The first condition or characteristic of a "good" or "legitimate" hypothesis is that it must be *based upon facts*. Here we use the word "fact" in the wide sense of anything ascertained to take place, or "to be the case," in Nature. The supposition is suggested only because it is a possible explanation of the facts. It is not created by the scientific imagination "out of nothing." It is intimately dependent on the suggestions of accurate experiment

and observation, and also on whatever knowledge the investigator already possesses. And as in its origin the hypothesis depends upon facts, so for its verification we must examine the relevant facts with the most rigorous exactness; and if there is any discrepancy, the hypothesis must be rejected or modified.

(2) A second requirement of a "good" hypothesis is usually stated in this way: it must not "conflict" with "received" or "accepted" knowledge, or with any of the "established" laws of Nature. In order to estimate the value of this statement, we must know exactly what it means. The hypothesis may be new and strange, and in that sense may conflict with the apparent implications of knowledge assumed to be established, and yet may be legitimate. It is legitimate if, when we consider both what the hypothesis implies and what is implied by our previously accepted knowledge, the discrepancy can be removed by mutual modification. For example, the assumption of the "antipodes" was once supposed to be in conflict with ordinary experience, for it seemed to mean that on the other side of the earth were people living with their heads "downwards." The difficulty was removed when it was understood that "down" means only the direction in which the mass of the earth attracts bodies by gravitation; and this direction is always in a straight line to the earth's centre. In this case, there was a modification of the "received knowledge" by which it was set in a new light.

An extreme case, in illustration of what we have said above, is found in the Ptolemaic theory of the solar system, which was based on the assumption (apparently warranted by the evidence of our senses) that the earth is the centre of the universe, and that the sun, moon, planets, and stars move round it. The hypothesis of Copernicus, that the sun is the centre of the solar system, simply

reversed this fundamental assumption. In reference to the Copernican hypothesis, Galileo said: " I cannot too much admire the intellectual power of those men who have accepted it and hold it to be true, and who in the vigour of their judgment have done such violence to their senses, that they have been able to prefer what their reason dictated to them, against what the evidence of their senses plainly represented to the contrary."

He adds that he can set no limits to his admiration of the way in which their reason has committed " such a rape upon their senses " as to become " mistress of their [the senses'] credulity." But Galileo knew that the " rape " was itself justified by more accurate sense-perception.

In this connection it is sometimes said that a legitimate hypothesis must be " conceivable." If " conceivable " means " not self-contradictory," there is no need to make a rule that the hypothesis must not contradict itself. If, on the other hand, " conceivable " means " easy to *imagine*," in the sense of making a mental picture of its meaning, then it is not true to say that a legitimate hypothesis must be " conceivable." Many things are not easy to *imagine*, although we know that they are true. It is not easy to *imagine* that if an organ were played by machinery in a hall, and no living creature were in or near it, the organ would make no *sound*. It is not easy to *imagine* that the earth is a globe, whose circumference is twenty-five thousand miles, revolving daily on its own axis, and annually in a vast orbit round the sun. It is not easy to *imagine* that the sun is a million times larger than the earth.

(3) All that is valuable in the demand that a hypothesis shall not conflict with laws of Nature believed to be established, is provided for in another rule: a legitimate hypothesis must provide a *basis for deductive inference* of consequences. This implies not only that the hypothesis must be clearly and distinctly conceived in itself; it must be conceived after the analogy of some agency of which we

have had experience. To assume something *utterly unlike* all that we are previously acquainted with, is to assume what can be neither proved nor disproved, for we could not draw any conclusions from it. Such an assumption is called a **barren hypothesis.** The " catastrophic " theory, formerly held by many students of Geology, if taken as a basis of investigation, assumes the operation of incalculable forces, the assumption of which leads and can lead to nothing. It is a barren hypothesis. Instead of these mysterious agencies, Lyell assumed that causes similar to those with which we are now acquainted have been acting uniformly for long ages. The nature of the causes at work being known, it became possible to calculate the nature of the effects, and thus reduce the facts of Geology to order and system. Or, again, to take, this time, an illustration which may seem fantastic; suppose it were assumed that the irregularities in the orbit of the planet Neptune were due to the arbitrary interferences of a demon: we have no knowledge of " demons " which would enable us to deduce from the hypothesis consequences which could be compared with the facts.

A *barren* hypothesis is illegitimate because it cannot be verified. It must be distinguished from a *false or erroneous* hypothesis. An erroneous hypothesis, such as the assumption that the speed of freely falling bodies varied with their weight, is an unsuccessful hypothesis; it can be developed and tested, but it fails to fit the facts. At the same time an erroneous hypothesis is not necessarily worthless. Before breaking down it may have done service to science; but if so, it is not the distinctively erroneous factors in the hypothesis which are serviceable. The assumption as to the speed of falling bodies, held by the contemporaries of Galileo, was simply false, and seems to have been of no service at all. It was a mere obstruction in the way of

discovery. But the Ptolemaic theory, that the earth is the centre of the solar system, had led to an accumulation of astronomical observations which had been gradually built up during many centuries, and had acted as a motive and inducement to discovery.

It has been justly said that the Ptolemaic theory, with its "geocentric" principle and its assumption that all the movements of the planets were uniform and circular, formed a definite steadying-point for astronomical science; but the service thus rendered was due to the systematic way in which the hypothesis was developed, and not to what was false in the theory. Moreover, in so far as it assumed that the earth was the centre from which all astronomical *observations* must be made, it assumed what all other astronomical theories have been obliged to acquiesce in The quality of the *observations* made at that period was not influenced in any way by the distinctive character of the Ptolemaic theory. Whatever the astronomical theory may be, the earth remains, for our perception, the *apparent* centre of the universe. But in so far as the hypothesis that the earth was the *real* centre of the universe was brought into play, and the apparent movements of the planets treated as real movements, the only utility of the error lay in its eventually refuting itself through the bewildering complexities to which it gave rise.

(4) The final characteristic of a legitimate hypothesis is that its deduced consequences must agree with the facts of Nature. This condition is one which we have already illustrated. And when the consequences of the hypothesis, deduced as accurately as possible, are confirmed by the results of accurate observation, then the hypothesis is said to be *verified*. We must distinguish *verification* from *proof*. Verification, which is logically an argument from the affirmation of the consequent in a hypothetical syllogism, is not proof unless we can show, or have good reason for believing, that no other hypothesis will explain the facts in question. How far this is the case will depend entirely on the particular investigation itself and the general character of our knowledge of its circumstances. Thus, when

Kepler, after trying nearly twenty different mathematical curves, found that an elliptical orbit could pass through all the different positions in which the planet Mars had been observed, he was reasonably justified in taking this hypothesis as not only *verified* but *proved*. We must add that the demand for absolute and conclusive *proof* of a hypothesis is an ideal demand. It assumes conditions which may be more and more realised, but which no hypothesis as yet realises.

The results of our discussion may be summarily stated thus. If a hypothesis is to claim recognition as a "good" or "legitimate" one, it must satisfy the following conditions:—

(*a*) It must be based upon events actually occurring in nature.

(*b*) It must be capable of being brought into accord with existing knowledge, by mutual modification if necessary.

(*c*) It must be capable of use as a principle from which consequences may be derived by deduction.

(*d*) The consequences so derived must be such as can be tested by experimental or other appeal to the facts.

We have shown how even a hypothesis which contains fundamental errors may be scientifically useful. We have now to see that a hypothesis which is not verified, or insufficiently verified, may be scientifically useful. If it is suggested by real knowledge of the facts dealt with, it will open up new and important lines of inquiry, in addition to providing a conception of the facts under which they may be conveniently collected, in thought, and harmonised. A hypothesis which is thus *fruitful*, but insufficiently verified, is called a **working hypothesis.**

The reader should notice the ambiguities of the terms "fact" and "theory." The term "fact" is frequently used, as we have used it in this book, to signify an event which takes place or is the case in Nature; and, in contrast, the term "theory" is used for an hypothesis which is suggested but not yet sufficiently verified. Some writers restrict the meaning of the term "theory" to "hypotheses which are fully established"; but, none the less, when a hypothesis is thus established, not, indeed, with absolutely conclusive proof, but established beyond the possibility of reasonable doubt, we tend to speak of it as a "fact." These ambiguities should be borne in mind.

We sometimes find the word "theory" being used in a depreciatory sense. Professor A. Wolf has well stated the principal source of this. "Deductive reasoning," he says, "is of course sound enough as far as it goes; there is nothing intrinsically wrong with it. But it is liable to be too abstract, in the sense of not taking into account all the factors involved. And it is just this that gives what little justification there is for the hackneyed *dictum* of 'men of practical affairs,' that this or that may be all right in *theory*, but will not work in *practice*. Except in purely hypothetical cases, what is true in theory is meant to be true in practice. But deductive theory is liable to overlook factors whose actual influence is in no way diminished by being forgotten." "Some of the Malthusians," says Professor Wolf in illustration, "relying on inadequate deductions, arrived at pessimistic conclusions about the future of the working classes. It was argued that any improvement in the regularity and amount of their wages would only encourage them to have still larger families, whose additional needs would continue to keep them at the poverty line. But the subsequent investigations of Charles Booth showed clearly that, as a matter of fact, the families of the

working classes steadily diminished in numbers, and their standard of life steadily became higher, as their income improved in amount and in regularity." This is one of the numerous cases where purely deductive reasoning stands in need of inductive testing and confirmation.[1]

8. Scientific Explanation

We have said that an essential part of the function or purpose of a good hypothesis is to "explain" facts; and, for the time being, we left this important word "explain" without further comment. We must now ask, in what sense is this word to be understood, when it is used in science, and how is its scientific meaning related to the meaning or meanings given to it in ordinary speech?

To "explain" a fact or causal law is to show its *connection* with other facts or laws and with previous knowledge. For example, the purpose of a hypothesis may be to *explain facts by a law,* as when many different and (at first sight) disconnected facts are shown to be instances of the same general law; or the purpose of the hypothesis may be to *explain a law by a more general law* or by other laws. In both cases we bring the "particular" or less general under the "universal" or more general.

(1) Science endeavours to explain *fact by law.* The "fact" may be a particular event, even a unique event; but the endeavour is made to bring it under general principles already known. For example: "An oculist may explain the common fact that short-sighted persons grow longer-sighted as they grow older, by showing how clear vision depends on focusing all the rays proceeding from each several point precisely upon [a minute area in]

[1] See A. Wolf, *Textbook of Logic*, Ch. XXI., § 4, especially pp. 245-6.

the surface of the retina; in short-sighted persons, the curvature of the lens of the eye is excessive, and therefore objects have to be nearer than would normally be necessary, in order that the rays proceeding from any point in them may be focused on the retina and not in front of it; but the curvature of the lens is maintained by certain muscles, which relax with age, and therefore, as years advance, clear vision of objects is possible at a greater distance." If he were called upon to explain some unique peculiarity of vision in a particular patient, the task would still be of the same kind; the facts to be taken into account would partly be facts peculiar to this case, and their consequences could be traced according to general principles.[1]

A famous case of the explanation of "fact by law" is seen in Kepler's discovery that the orbit of the planet Mars is elliptical, not circular. (It should be noted that Kepler was born in 1571 and died in 1630, and that his predecessor, Tycho Brahe, was born in 1546 and died in 1601.) The observations of Tycho Brahe had determined a great number of successive positions of the planet Mars to a high degree of accuracy; and the resulting orbit appeared to be extremely irregular But the earth itself, from which the observations were made, is in motion round the sun; hence it was necessary to distinguish that part of the irregularity of the orbit of Mars which was due to the earth's motion, and then to ascertain what curve corresponded to the true positions of the planet. He assumed the earth's motion to be circular, which is approximately true; but the orbit of Mars was evidently not circular The picture which Kepler presents to us of the working of his own mind while pursuing this research is full of the most intense interest. It would be impossible, without entering into mathematical details, to explain the process by which the ultimate suggestion was brought under his consideration; and it would be equally so to convey an idea of the immense mass of calculation through which he toiled in putting each of his successive theories to the test of agreement with the observations. Finally, after working his way in alternate exultation at anticipated triumphs, and bitter disappointment when, one after another, they vanished in air—driving him, as he says, "almost to insanity"—he at length had the intense gratification of finding

[1] For this example I am indebted to Joseph, *Introduction to Logic*, second edition, Ch. XXIII., pp. 507-8.

that *an elliptic orbit described about the sun in one of the foci* agreed accurately with the observed motions of the planet Mars. The irregularity of its movement vanished; all its observed positions became intelligible, were "explained," when seen to be successive points on this simple and symmetrical curve. The hypothesis, thus verified for Mars, was extended by analogy to the other known planets, and was verified in their movements also, by observations as accurate as were then available—Kepler perceiving that his original assumption as to the motion of the earth was only an approximate one. Thus was established "Kepler's First Law." There could be no better example of how disconnected facts are explained by being brought under a general law.

(2) Science also endeavours to explain "law by law." A hypothesis is sought for (and formulated by the prepared mind) *from which* a law, already obtained by previous induction but standing in no apparent relation to other laws, may be obtained by deductive reasoning. A law "standing in no apparent relation to other laws" is what we have called an "empirical law" (see page 243). Examples abound in the history of science and in contemporary investigation. The following is an illustration of this kind of explanation and of the lack of it. The regular disintegration of certain kinds of rock is largely due to frost succeeding rain, because the water in the crevices expands when freezing: although this peculiar property of freezing water is itself a law as yet unexplained.

A very famous example of the explanation of "law by law" is seen in Newton's derivation of Kepler's laws of planetary motion from the law of Gravitation. Kepler, by induction from an immense number of astronomical observations made by Tycho Brahe and himself, had discovered three fundamental laws of planetary motion (we have already quoted the first of them). The three laws are these: (*a*) that the planets move in elliptical orbits having the sun in one of their foci; (*b*) that the velocity of the moving planet is such that an imaginary line joining it to

the sun sweeps out equal areas in equal times; and (*c*) that the squares of the times which any two planets take to complete their revolutions round the sun are proportional to the cubes of their mean distances from the sun. Newton showed that these laws can all be deduced from the law that any two bodies attract each other with a "force" varying inversely as the square of the distance between them, and directly as the product of their masses.[1] This is the Newtonian formula for the law of Gravitation, which has been slightly modified in recent years through the investigations of Einstein.

When we look back over the various types of direct and indirect induction which we have described and illustrated, we realise that we must not expect too much of science, or read too much into the word "explanation." The truth is that to *explain* a "fact," in science, comes in the last resort to this—that we show it to be part of a wider "fact": in direct Induction, by finding its connections with antecedent or co-existing events, and in indirect Induction, by finding a common nature or common principle in the events thus connected. In this sense, *all Induction is explanation, and both are generalisation.* As explanation advances, events which differ widely in many ways are found to be in some of their aspects expressions of the same principle—the fall of a stone to the earth, the motion of the moon, and the orbit of a distant planet round the sun. The *interdependence* of events becomes more evident, and not one event but many are capable of the same explanation. Every hypothesis is an attempt at explanation; every established theory is an explanation of wider or narrower

[1] For a short explanatory statement of the circumstances which led to this famous discovery, see Mellone, *Introductory Textbook of Logic*, Ch. IX., pp. 328 ff., and Joseph, *Introduction to Logic*, Ch. XXIII., pp. 513 ff.

scope. And in this same sense we may say that Induction is the ordering of facts into a *system* (see Chapter VI., page 105).

From scientific explanation, so understood, we must carefully distinguish what may be called "popular" explanation, which includes "explanation" addressed to an individual mind, as in the case of the teacher "explaining" to the pupil. The teacher "looks to the individual mind, and seeks to bring the new event into relation with something already there; the explanation will vary in character with the person to whom it is addressed, and in many cases will be utterly inadequate from the scientific point of view; it may merely be a rough analogy or illustration, as is constantly the case with the only explanations which can be given to children." In general, "popular" describes the unknown and unfamiliar as being made up of the known and the familiar. Thus, in the case of a *gas*, we know that pressure is proportional to density. The atomic theory is applied to explain the concomitant variation, and "popularly" in some such way as this: a gas is made up of a vast number of minute particles always flying about and striking against one another and bounding off again; greater "density" means that the particles have a smaller space in which to fly about and strike one another, and consequently do it more quickly and more often; and the result is an increase of the pressure of the gas on the sides of the containing vessel.[1]

Finally a word of explanation is needed over the word "relevant" in reference to the circumstances of an event which is under inductive investigation. Every event is a detail of the world and the world itself is a detail of the

[1] See W. K. Clifford, *Lectures and Essays*, pp. 101-3.

universe; and there is no reason for denying that every event has effects throughout all space and time. It scarcely needs to be said that all such considerations are "irrelevant" in any particular investigation. The vast majority of the circumstances of an event can be, and have to be, *ignored* in any special study of the event in question. On the other hand, our whole account of the Methods of direct Induction has shown that, in a more special sense of the word, they are entirely concerned with what is "relevant" and what is not. Previous knowledge, experience, and training enable the investigator to set limits to what we may call the "field of relevance" in any case under his consideration; and direct Induction then enables him to eliminate what is "irrelevant" because not causally connected with the event which he is studying. The Methods of direct Induction have been described as "weapons of elimination"; what can be "eliminated" from the circumstances of an event without affecting the event is not causally connected with it. This principle is of special importance in the Method of Double Agreement, the Method of Difference, and the Joint Method of Difference and Agreement.

9. Historical Illustrations

In this section we shall give, from the history of science, some detailed illustrations showing (1) the importance of the mental factor in scientific observation; (2) the displacement of the Ptolemaic by the Copernican theory of the solar system owing to the accumulation of more accurate observations; (3) the meaning of the "abstract" character of modern physical science.[1]

[1] Readers who do not intend to carry their study of the subject beyond the elementary stage, may omit this section.

(1) The history of Logic, from the time of Bacon and Galileo downwards, was mainly determined by the endeavour to secure what seemed to be a great reformation in science, namely, the abandonment of all preoccupation with deductive inference from premisses accepted on authority, and concentration on the facts of experience. Logic was no longer to be the instrument of a consistent elaboration of dogma, but the instrument of a true explanation of Nature. Francis Bacon advocated this principle of investigation with prophetic fervour. The facts were to be allowed to speak for themselves, while the mind maintained a purely passive or receptive attitude. No conjectures or hypotheses were to be made; we were to wait until our systematic records of the facts revealed the general principle embodied in them. The method of science was to be that of statistical tabulation, so compiled and arranged as to force upon the methodical collector of observations the laws which the facts require to explain them. Hence Bacon said, assuming that his methods of statistical tabulation were understood, "our method leaves not much to acumen and strength of wit, but nearly levels all wits and intellects." He seems really to have believed that a scientific method could be devised so mechanical and "fool-proof" that anybody could work it.

The value of this doctrine lies in its insistence on the need of a laborious, impartial, and unprejudiced collection of facts. The error of the doctrine lies in ignoring what we have already seen (page 216) to be the most important characteristic of observation. The mind cannot collect facts methodically unless it also *selects* them. The mind must go to meet the facts armed with ideas. Prior to any "collecting" the mind must know for what purpose the collection is to be made. And during the collection of material, it must be ready to take up any indication of a

law embodied in the material, and make of it a hypothesis which can be tested.

The work of Charles Darwin affords a unique illustration of what we have just said. He seems to have been a born collector of facts: "[At eight years of age] my taste for natural history, and more especially for collecting, was well-developed. I tried to make out the names of plants, and collected all sorts of things—shells, seals, coins, minerals. The passion for collecting which leads a man to be a systematic naturalist . . . was very strong in me, and was clearly innate, as none of my sisters or brothers ever had this taste." During his student years at Cambridge, and most of all during the voyage of the *Beagle,* these pursuits were followed up with eager enthusiasm and laborious thoroughness. The result, as regards the great problem with which his mind was chiefly occupied, is thus set forth in his own words.

"After my return to England, it appeared to me that . . . by collecting all the facts which bore in any way on the variation of animals and plants under domestication and in Nature, some light might perhaps be thrown on the whole subject . . . I worked on true Baconian principles, and without any theory collected facts on a wholesale scale,[1] more especially in relation to domesticated productions, by printed inquiries, by conversation with skilful breeders and gardeners, and by extensive reading . . . I soon perceived that *selection* was the keynote of man's success in making useful races of animals and plants. But how selection could be applied to organisms living in a state of Nature remained for some time a mystery to me. In October 1838 . . . I happened to read for amusement

[1] "Without any theory" but always in view of their relation to the question which was taking more and more definite form in his mind.

Malthus on *Population*, and, being well-prepared, from long-continued observation of the habits of animals and plants, to appreciate the struggle for existence which everywhere goes on, it at once struck me that under these circumstances, favourable variations would tend to be preserved, and unfavourable ones to be destroyed. The result of this would be the formation of new species. Here, then, I had at last got a theory with which to work."

Although Darwin says that he worked "on true Baconian principles," it is quite clear that he was entirely free from the one-sided bias of the Baconian method, while at the same time he was always ready to abandon or to modify even the most cherished hypothesis if it failed to fit the facts. "I have steadily endeavoured," he says, "to keep my mind free, so as to give up any hypothesis, however much beloved (and I cannot resist forming one on every subject) as soon as facts are shown to be opposed to it. Indeed, I have had no choice but to act in this manner; for, with the exception of the coral reefs, I cannot remember a single first-formed hypothesis which had not after a time to be given up or greatly modified." What Francis Darwin says of his father is true of the scientific genius in every branch of inquiry: "He (Charles Darwin) often said that no one could be a good observer unless he were an active theoriser; and this brings one back to what I said about his instinct for arresting exceptions. It was as though he were charged with theorising power ready to flow into any channel on the slightest disturbance; so that no fact, however small, could avoid releasing a stream of theory, and thus the fact became magnified into importance."[1]

The contrast between Bacon's view of what is involved in scientific observation, and Darwin's view, is an instructive

[1] The quotations are from the *Life and Letters of Charles Darwin*, edited by Francis Darwin, 1887.

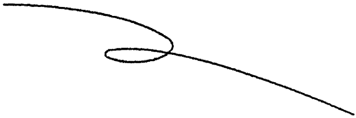
illustration of facts which we have already emphasised (see page 217). Bacon's error was psychological as well as logical. Sense-perception is a mental as well as a physical activity; and observation, like ordinary perception, is selective. In order to observe, not only must the attention take a particular direction, but we must be more or less aware of what we are looking for. In science, the true observer brings to his observation more than he finds in it.

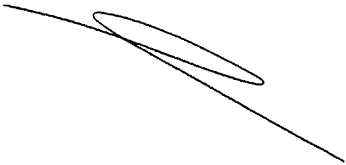
(2) In the next place, we shall describe the displacement of the Ptolemaic by the Copernican theory in the seventeenth century. We see a famous hypothesis, with the authority of centuries behind it, and apparently warranted by the most evident facts of sense-perception, gradually breaking down under the weight of its own complexities, when faced with an accumulation of more extensive and more accurate observations.

In the second century of our era, the Alexandrian astronomer Claudius Ptolemaeus ("Ptolemy") devised a hypothesis to explain all the observed movements of the heavenly bodies in one system. The fundamental assumption with which he started was that the earth is *at rest*—the one fixed centre of the universe; and the movements of the sun, planets, and stars, which our senses show us, are their *real* movements. Any other view was dismissed by Ptolemy as "absurd." A second assumption, derived mainly from Aristotle, was that the heavenly bodies move in circles, because the circle is "the most perfect figure." Now it had long been known that the apparent movements of the planets in the sky are far from simple. A planet watched night after night would be seen first to move from west to east at a varying rate, then to stop, then to adopt a retrograde movement, then to stop again, then to go

forward again, and so on. The following diagrams[1] show two typical cases of this apparent "retrograde" motion; the path indicated by the curved line in each case occupying six months. The upper diagram shows a portion of the apparent path of Venus; the lower one, of Mars; and the horizontal line represents the celestial equator.

Ptolemy's problem was to analyse all these movements into a system of circular movements; the *earth at rest* being the centre of the whole system. On the evidence of our senses—as exemplified in the diagrams—we know that the planets do not move in simple circles round the earth. Ptolemy therefore found it necessary to invent a system of *epicycles*, an "epicycle" being a smaller circle whose centre moves along the circumference of a greater. Each planet he supposed to move uniformly round the circumference of the epicycle, while the centre of the epicycle moved uniformly in a circular orbit round the earth. The actual path which would be traced out by each planet, under these circumstances, is shown on following page.



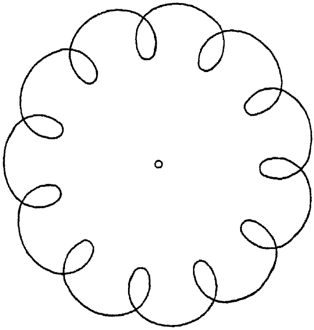


Even this amount of complication failed to make the theory fit the observed facts; and as the years went on,

[1] After Sir O. Lodge, *Pioneers of Science*, pp. 21, 22.

and astronomical observations became more numerous and more accurate, the supporters of the Ptolemaic system were obliged to make it more and more complicated. Even if it had remained without a rival, it would at length have broken down through its own increasing complexity.

Nevertheless the Ptolemaic theory went far to satisfy the conditions of a legitimate hypothesis. Its author was fully aware of the assumptions which he was making; and we may admit (*a*) that his hypothesis was based directly upon the observed facts, as they were known during his life time; (*b*) that it did not assume the existence of anything which could not be observed, or of any mysterious or occult agency; (*c*) that it was put forward as a hypothesis capable of reducing all the observed movements of the planets to an orderly system, and that its consequences, derived by mathematical calculation, could be tested by the facts; (*d*) that it did almost reduce the observed movements (as they were then known) to an orderly system. What happened was this: the facts were not so much disproving it as continually going beyond it and requiring new complications to make it cover them. It was superseded only when a hypothesis equally legitimate but mathematically *far more simple* was presented as accounting for the observed facts.



Such a theory was formulated by Copernicus,[1] who reversed the fundamental assumption of the Ptolemaic system, and assumed that the sun is the centre of the system of planets which all move round it, so that the earth is only one planet among the others, and the observed movements of the planets are *apparent*, not *real*. He proved mathematically that his "heliocentric" theory accounted for the apparent movements of the planets much more simply and satisfactorily than the "geocentric" theory could ever do. The apparent "retrograde" motions of the planets are due to the fact that the earth, from which the observations are made, is itself moving round the sun. He could not account for all the facts because he still assumed that the planets moved in perfect circles. And he followed his predecessors in believing that the stars were fixed in a great all-embracing sphere; but as their apparent positions (in relation to one another) did not vary even from the most widely distant points of the earth's orbit, he concluded that their distance from the solar system must be immensely greater than astronomers had previously supposed it to be. This true conclusion was taken by some of his contemporaries to be an objection to his system; they regarded it as "inconceivable" that the stars should be at such immense distances.

The whole story of the victory of the Copernican theory over the Ptolemaic affords an extraordinarily vivid illustration of what "science" really is, and of the way in which an inadequate hypothesis is displaced by a better one.

[1] Ptolemy died at Alexandria towards the end of the second century A.D. Copernicus died in May 1543; and his book *De Orbium Celestium Revolutionibus*, the result of more than twenty years of labour, was published on the day of his death.

(3) Our third series of illustrations is intended to suggest an elementary conception of *the limitations of our senses,* in the case of vision, and of the way in which physical science is becoming more and more *abstract* in the sense of moving further and further away from the apparent facts of sense-perception. Take the case of the Law of Gravitation. Galileo by actual experiment discovered the laws of falling bodies, when they fall in the neighbourhood of the earth's surface. These laws were general statements of concrete fact, and as little abstract as such statements could be. Meanwhile, enormous masses of observations of the movements of the planets had been accumulated and tabulated by Tycho Brahe; and Kepler, calculating on the basis of these observations, formulated his three laws of the planetary orbits (see page 257). These again, though expressed in mathematical terms, were general statements of the least abstract kind. Then Newton collected together Kepler's laws, and Galileo's law of falling bodies, and the laws of the tides, and what was known of the laws of the comets, in one law which embraced them all—a law of wider generality and of a correspondingly higher degree of abstractness—the law of gravitation; and Newton's law showed not merely why the previous laws were right, but why they were not quite right.

Recently Einstein has done with Newton's law what Newton had done with previous laws; and in like manner he has shown that Newton's law is "not quite right." He has generalised it, and shown that the Law of Gravitation is not a mechanical but a geometrical law, and that Gravitation is a consequence of the nature of Space. Its treatment has become a department of abstract mathematics. The fact is that the power of mathematical technique has developed to an extent which only the initiated can appreciate. It can deal, at the highest stage

of abstractness, with the most general relations involved in the material universe, moving in a region apparently out of touch with all the concrete facts which embody those relations; and at the end of the demonstration, it can produce *results verifiable by observation.*

In the next place, take the case of those sciences which investigate the "constitution of matter." What has happened here may be pictorially illustrated thus. Imagine a simple diagram drawn as follows: an upright band of moderate width (its "width" is irrelevant to the illustration), supposed to be continued indefinitely above and below, coloured black throughout except where in its middle region it is crossed by a thin horizontal line of white. The white line represents the portion of the material universe which is directly revealed to our senses; the extent of the "band," above and below, represents the material for which we have no special sense-organ. Everything which has been discovered of the energies belonging to these regions has been discovered not because they affect any sense-organs of ours, but because instruments have been constructed delicate enough to manifest some of their effects.

In terms of present knowledge, a large part of the "band" represents the range of the electro-magnetic radiations (undulations) which are known actually to constitute the material of the physical universe. They range from wave-lengths of unimaginable minuteness at one extreme to those of enormous length at the other. We cannot mentally realise the figures which are involved. Remembering that a "centimetre" is the hundredth part of a metre, we divide a centimetre into three thousand million equal parts, and we find that one of these represents the wave-length at the lower limit of measurable

radiation. When the wave-length has increased to a twenty-five-thousandth of a centimetre, the undulation begins to affect the retina, producing a visual sensation of darkest violet; and when it has increased to twice this length, it ceases to affect the retina visually, after producing a sensation of dullest red. Thus, " visible light " is only one special case of a kind of undulatory radiation which pervades a material universe. The "X-Rays," turned to account in medical work, form a series which, in our imaginary diagram, would fall below the visible series, since their wave-lengths are much shorter than the latter; and the radiations employed in " Wireless," whose wave-lengths are very much longer, would form a series standing far above the visible series. In this region, we are dealing with movements which are essentially intangible, invisible, unimaginable; but they are mathematically conceivable. In becoming more abstract, science becomes more mathematical; and this means that the scientific statement of a law of Nature becomes a mathematical formula Yet notwithstanding its assumptions and its abstract character, the investigation of a problem in mathematical Physics starts from observation and experiment—that is, at bottom, from sense-perception—and its final results must be capable of being tested by sense-perception.

Exercise VI

1. What do you understand by the term " Induction " ? State clearly the aim of " Inductive Logic."

2. Have the inductive and deductive processes of reasoning anything in common ? Give reasons for your answer.

3. " Induction is the discovery of Major Premisses." Explain and discuss this statement.

4. Explain and illustrate the terms: (*a*) Antecedent and consequent, (*b*) Condition, (*c*) Occasion, (*d*) Cause, (*e*) Law of Nature.
Is every " antecedent " a " cause " ?

5. (*a*) Define precisely what Science understands by the Cause of an event, and illustrate your answer by a typical example of Causation determined by experiment

(*b*) What is the relation of the scientific conception of Cause to the conception of Cause as the "sum-total of conditions" on which the event depends ?

(*c*) Can you account for the divergence between the scientific and the popular view of the Cause ?

6. In what sense may it be affirmed, and in what other sense may it be denied, that "an event can only have one cause" ?

7 Carefully explain the term "Uniformities of Co-existence." How far can such uniformities be distinguished from "Uniformities of Causation" ?

8. "Induction by simple enumeration can never lead to more than an Empirical Law" Explain and discuss this statement.

9. Show how hypotheses of causation may be suggested, (*a*) by Enumeration, (*b*) by Analogy.

10. What place does *hypothesis* hold in the Methods of Direct Induction ? Illustrate your answer.

11. "Theory [*i.e.* Hypothesis] is the General, and Experiments are the Soldiers" (Leonardo da Vinci). Discuss the value of this Analogy.

12. On what does the strength of an *argument from* Analogy depend ? Comment on the following extracts:—

(*a*) "I am a Jew. Hath not a Jew eyes ? hath not a Jew hands, organs, dimensions, senses, affections, passions ? fed with the same food, hurt with the same weapons, subject to the same diseases, healed by the same means, warmed and cooled by the same summer and winter, as a Christian is ? If you prick us, do we not bleed ? If you tickle us, do we not laugh ? if you poison us, do we not die ? and if you wrong us, shall we not revenge ? if we are like you in the rest, we will resemble you in that" (*Merchant of Venice*, Act III., Scene i.).

(*b*) "I have somewhere read . . . of a most fatal battle which was fought by two opposing armies, in which almost all the combatants on both sides were killed, 'because,' says the historian, 'though they had offensive weapons on both sides, they had none for defence.' So, in this war of words, if we are to use the only offensive weapons, if we are to indulge in invective and abuse, the contest must be eternal" (C. J. Fox, *Speech on the French Overtures for Peace, February* 1800).

(*c*) "The Whale resembles the Shark not only in all the common characteristics of Vertebrates, but also in its submarine habitat and in being (as regards many species) one of the very largest of marine animals. Like the shark, it is fish-like in external form, its fusiform body being well adapted for cleaving the water. Anteriorly its body passes into the head without any distinct neck, and posteriorly it is furnished with a swimming-tail into which the body gradually tapers. It has no hairy covering Like the shark, again, it has a wide mouth, and is of predacious habits, feeding only on living animal nutriment. Therefore we may with great probability conclude that its method of respiration is like that of the shark—*i.e.* that it breathes the oxygen dissolved in the water, and has no need to be supplied with atmospheric air."[1]

13. Examine the following:—

(*a*) "States must decay, as individuals do" (see Mill, Bk. V., Ch. v., § 6).

(*b*) "Smith and Brown resemble each other in possessing a certain kind of skill; they can both make locks Smith is further given to picking them with a view to robbery. Therefore Brown is also addicted to lock-picking with a view to robbery."

14. How does Experiment (*a*) resemble, (*b*) differ from, Observation? What is a "natural experiment"?

15. "The use of scientific instruments partakes of the nature both of Experiment and Observation." How far is this true?

16. "No theorising apart from observation, and no observing save in the light of theory" Comment on this.

17. To what extent is it true that "sciences which are mainly dependent on Observation are at a great disadvantage compared with those in which Experiment can be largely employed"?

18. How does the Method of Single Agreement differ from simple enumeration? Can it ever prove a particular causal connection? If not, why not?

19. Give some examples of simple experiments fulfilling completely the conditions of the Method of Single Difference, and point out how they do so.

20. Compare and contrast (*a*) the method of Double Agreement and the Method of Single Agreement, and (*b*) the Joint Method of

[1] See Boyce Gibson, *The Problem of Logic*, 1908, pp. 359-60.

Difference and Agreement and the Method of Single Difference, showing the advantages of the first Method in each case.

21. Explain concisely the Method of Concomitant Variations, and give two examples of its use. What is its relation (*a*) to the Method of Single Difference, (*b*) to that of Single Agreement?

22. Explain the Method of Residues When can it prove that one event is the cause of another, and when can it only suggest an inquiry into causation?

23. What is meant by "Explanation" in science? Can we distinguish different kinds of "Explanation"?

Carefully examine the following statement. "Science never *explains;* she only reduces complex events to simple ones of the same kind, as when she deals with certain phenomena of Magnetism by supposing every ultimate particle of the substance to act as a magnet."

24 Carefully explain the essential steps in the Method of Indirect Induction, with special reference to the part played by hypothesis in this Method.

25 Explain and illustrate the conditions to which a legitimate or "good" hypothesis must conform

Distinguish between a working hypothesis and an established hypothesis.

26. Select any one of the great conclusions of modern science, and trace the process from the formation of a hypothesis to the attainment of a reliable conclusion What can be said as to the *origin* of the hypothesis in the case you select for analysis?

27. What is the logical character of the appeal to "the testimony of the senses"? Does this testimony, for example, really bear witness to the motion of the sun relatively to the earth?

28. Show how the following rule for personal investigation contributed to successful discovery: "I had, during many years, followed the golden rule—namely, whenever a published fact, a new observation, or a thought came upon me, which was opposed to my general results, to make a memorandum of it without fail and at once; for I had found by experience that such facts and thoughts were far more apt to escape from the memory than favourable ones"

29. Comment on any points of *logical* interest in the following:—

(*a*) Einstein's assumptions led him to the conclusion that light from a star, passing near the sun, would be deflected This has been confirmed by measurements made by various eclipse expeditions

(during a solar eclipse, stars nearly in the same line of sight as the sun can be seen).

(*b*) If the evolutionary hypothesis is true, geologists should find a gradual increase in the complexity of fossil remains as they pass up through the various strata of the earth's surface; and this is found to be the case.

(*c*) A bank cashier discovers the disappearance gradually of small sums of money. Circumstances lead him to suspect clerk A He makes opportunity for theft, and finds that A has taken a small sum of money

(*d*) In 1628 William Harvey, on the basis of observations and experiments, announced his discovery of the circulation of the blood by means of the heart; but he had not obtained complete ocular demonstration. Sixty years later the Dutch microscopist, Anthony van Leuwenhok, tried various animals in attempts to see the circulation—the ear of a white rabbit, the wing of a bat, etc Finally he directed his microscope to the tail of a tadpole He thus expresses himself: " A sight presented itself more delightful than any that my eyes had ever seen. While the animal lay quiet in the water, and I could bring it before my microscope as I desired, I saw not only that the blood was conveyed through exceedingly minute vessels from the middle of the tail towards the edges, but that each of the vessels had a curve or turning, and carried the blood back towards the middle of the tail, in order that it might be again conveyed to the heart. I then understood that the blood-vessels which I saw in the animal, and which are called arteries and veins, are one and the same, that is to say, they are properly termed arteries as long as they convey the blood to the furthest extremities, and veins when they bring it back to the heart."

(*e*) Five towns, of approximately the same size and similar in other conditions, in which there is a large percentage of crime, are compared with five other similar towns in which the percentage of crime is small. Possible causes are: size of police force, efficiency of public education, severity of penalties inflicted for crime, and presence or absence of licensed houses. It is ascertained that while every one of the other conditions is found in one or more members of each group, none of the towns with a small percentage of crime have licensed houses, while all of those with a high percentage have such houses. (This example presupposes conditions in the United States.)

(*f*) " Tell me, Protagoras," said Zeno, " does a single grain of millet make a noise in falling, or the thousandth part of a grain ? " And when Protagoras replied that it did not, Zeno asked, " Does a bushel of millet make a noise when it falls, or not ? " When Protagoras replied that it did, Zeno asked, " Is there not a *ratio* of

a bushel of millet to one grain and to the thousandth part of a grain ? " When he replied that there was, Zeno said, " Well then, will not the ratios of the *sounds* to one another be the same ? "

(*g*) " What is known as the ' ripple-mark ' in sandstone surfaces may be produced in various ways The most familiar way is by the action of the tides on the sand of the sea-shore; and the interpreter who knows only this would ascribe the sandstone-marks at once to this agency, operating at some distant geological period But ripple-marks are produced also by winds on drifting sands and by currents of water where no tidal influence is felt. Is it then impossible to decide between these alternative possibilities of causation ? No; wind-ripples and current-ripples and tidal-ripples have each their own special character and accompanying conditions; and the hypothesis of one rather than another may be made good by means of these."

The reader should carefully analyse examples of inductive reasoning which are not too technical. For further illustrations, reference may be made to Welton, *Manual of Logic*, Vol. II., Chapter V., § 154, pp. 122 ff (out of print, but accessible in some libraries); to E. M. Whetnall's edition of Welton and Monahan, *Intermediate Logic*, Chapter XXXI. (and extracts given in pp. 485 ff.); to A. Wolf, *Textbook of Logic*, pp. 357 ff.; to H. W. Joseph, *An Introductory Logic*, Chapter XX.; and, for an instructive account of some famous men of science and their methods, and of present-day tendencies, see Westaway, *Scientific Method*, Chapters XXVI. to XLV.

CHAPTER VIII

LOGICAL FALLACIES AND THEIR SOURCES

1. Fallacies in General

In the first chapter of this book we pointed out and briefly discussed the most important ways in which beliefs may be formed in the mind without conscious, deliberate, or explicit reasoning. It must not for one moment be supposed that beliefs so formed are for that reason *necessarily* infected with error; but it is evident that these non-logical sources of belief may and often do involve serious errors in the beliefs which they produce, and may lead to serious mistakes in reasoning.

By a " mistake in reasoning " we mean a violation of one or other of the principles on which the correctness or validity of reasoning depends. This is what is meant by a logical " fallacy." From this point of view, fallacies might be classified according to the logical principle violated; but the ways in which this may happen are so numerous that anything like a complete classification is out of the question. There are, however, certain kinds of invalid reasoning which are of frequent occurrence; and a descriptive analysis of them is of practical as well as of theoretical importance. In this sense we may speak of a " classification " of fallacies, without implying that the classification is complete or exhaustive. There is a traditional classification of logical fallacies which is of historic interest as going back to the teaching of Aristotle; and in an elementary work, it is most convenient to base our account on Aristotle's list.

The traditional doctrine is stated and illustrated in a remarkably interesting way by De Morgan in his *Formal Logic* (originally published in 1847; new edition, edited by Professor A. E. Taylor of

Edinburgh, 1920). For a short account of the Aristotelian classification, see Mellone, *Introductory Textbook of Logic*, Ch. X. An instructive attempt to classify fallacies according to the logical principle violated, will be found in Welton's *Manual of Logic*, Vol. II., Bk. VII (out of print), see also E. M Whetnall's edition (third) of Welton and Monahan, *An Intermediate Logic.* John Stuart Mill's classification, given in the Fifth Book of his *System of Logic*, is valuable and suggestive.

Aristotle's purpose, in his account of logical fallacies, was entirely practical—to describe the various types of false argument which occurred in current controversy. He divided them into two principal classes: (*a*) fallacies due directly to the misuse of language, and (*b*) those which arise from defects of thought rather than of language.

Under the first he enumerates fallacies usually called by the following English names:—

(1) " Equivocation " (Ambiguity of Terms);
(2) " Amphiboly " (Ambiguity of Grammatical Structure);
(3) ' Composition," and its converse " Division ";
(4) ' Accent ";
(5) ' Figure of Speech."

Under the second head, he enumerates fallacies known by the following English or Latin names.—

(1) "*Accidens* ";
(2) "*A dicto secundum quid ad dictum simpliciter*," and the converse;
(3) " Begging the Question " (*Petitio Principii*);
(4) " *Consequens* ";
(5) " Many Questions ";
(6) " *Ignoratio Elenchi* ";
(7) " *Non causa pro causa.*"

Of the preceding, we have already explained the fallacies called "Equivocation" (see page 127), "Composition," and "Division" (page 28), "*A dicto secundum quid*" and its converse (page 154). We proceed to explain "*Petitio Principii*" and "*Ignoratio Elenchi.*" The six remaining fallacies in Aristotle's list, "Amphiboly," "Accent," "Figure of Speech," "*Accidens,*" "*Consequens,*" "Many Questions," and "*Non causa pro causa*" are logically of less importance, and for these, the reader is referred to the Glossary.

(1) The term **petitio principii**, at any rate in its English equivalent, "begging the question," is one of the logical terms which has found its way into the language of ordinary life. It means "assuming the thing which is to be proved." This is also called "reasoning in a circle" (*circulus in probando*), coming round, by way of *conclusion*, to what has already been *assumed*, either expressly or by implication. We may distinguish two ways in which *petitio principii* may be committed.

(*a*) It is possible to commit this fallacy by direct assumption of the very proposition which is to be proved, and to do this even in a single step of inference; but in this gross form it is usually due to some ambiguity or confusion of language, or to the misuse of synonyms. The richer a language is in synonyms, the easier it is to fall into such errors: "The Bill before the House is well calculated to elevate the character of education in the country, for the general standard of instruction in all the schools will be raised by it." This is a case of "proving" a proposition expressed in concrete terms by the same proposition expressed in abstract terms. Again: "The two gases, oxygen and hydrogen, combine because of their mutual *chemical affinity.*" This is no argument, because "chemical

affinity" in this case is only an abstract name for the observed fact that the two gases do combine in the proportions determined by experiment. The immediate assumption of the proposition to be proved may be well concealed. De Morgan gives the following as an example of a *kind or type* of reasoning in a circle which is "almost of daily occurrence":—"Porcelain says to Crockery, 'I am better than you.' 'I don't see that,' says Crockery. 'Why,' says Porcelain, 'am I not made of better clay?' 'How so?' says Crockery. 'Why,' returns Porcelain, 'my clay makes Porcelain, and yours only makes Crockery.'" What usually happens is that the assumption of the proposition to be proved is separated by a greater or less interval from its statement as a conclusion, as in lengthy scientific or philosophical arguments.

On the other hand, the fallacy may be committed in a single word, by the use of what is called a "question-begging epithet (or name)." Most political catch-words, or names which can be given a rational meaning but are used as political catch-words, laudatory or (more usually) the reverse, are merely question-begging names, used to create an emotional atmosphere of approval or condemnation: "Taxing the people's food"; "Bolshevism"; "Anti-waste"; etc., etc. The fallacy is crudely but effectively summed up in the proverbial saying, "Give a dog a bad name, and hang him."

(*b*) The fallacy of *petitio principii* is also committed when we take for granted a general proposition or principle which involves the required conclusion, and which is just as much in need of proof as the conclusion itself; or indeed, when any general principle is falsely taken to be self-evident. There is an interesting example in the first chapter of Herbert Spencer's well-known book on *Education*.

After stating that "acquirement of every kind has two values—value as *knowledge* and value as *discipline*," he discusses the value of different subjects from the point of view of knowledge. He then turns to the disciplinary value of various studies, and proceeds as follows: "Having found what is best for the one end, *we have by implication found what is best for the other*. We may be quite sure that the acquirement of those classes of facts which are most useful for regulating conduct, involves a mental exercise best fitted for strengthening the faculties. It would be utterly contrary to the beautiful economy of Nature, if one kind of culture were needed for gaining information and another kind were needed as a mental gymnastic." Here the writer "proves" his conclusion (italicised in the quotation) by simply taking for granted a general principle which he is pleased to call "the beautiful economy of Nature." When we argue from general principles, these must either be admitted as self-evident, or they must be capable of proof independently of the conclusion which we wish to derive from them.

(2) The term **ignoratio elenchi,** as the name of a logical fallacy, means "ignoring the conclusion to be proved (or disproved)"—in other words, arguing to the wrong point or proving the wrong conclusion. In English it may be briefly called "Irrelevant Conclusion." It has been well said that "no fallacy is more common or more easily fallen into than this; any one who has had experience of disputations and debates knows how constantly recurrent is the tendency to wander from the real point at issue, especially when the subject under discussion is a wide one, and how necessary it is sometimes for a speaker to begin his remarks by reminding the disputants what the question under discussion really is." The tendency to evade the

point really at issue may be practically dangerous. De Morgan observes: "If a man were to sue another for debt, for goods sold and delivered; and if the defendant were to reply that he had paid for the goods furnished, and plaintiff were to reply that he could find no record of the payment in his books, the fallacy would be palpably committed. The reply, supposed true, shows that either the defendant has not paid, or that the plaintiff keeps negligent accounts. It is plaintiff's business to prove sale from what is in his books, not the absence of payment from what *is not* in his books; and it is defendant's business to prove payment from his vouchers."

To attempt to throw what is called the "burden of proof" on the wrong side is another kind of Irrelevant Conclusion. Proof of an assertion should generally be given by the person who makes that assertion. To endeavour to transfer to an opponent the task of proving the negative (the logical contrary or contradictory) if the assertion is an Irrelevant Conclusion. To prove a *bare denial* may be impossible; the required negative usually can only be proved by proving a positive which implies it. Accordingly, as De Morgan observes, when the two sides of a question are a positive and a negative, the burden of proof is generally considered to lie upon the person who is interested in maintaining the positive. Thus, in a Court of Law, the trial is opened by the case for the prosecution, and the *evidence for* the charge or the claim has to be presented first. If the charge is shown to imply the defendant's presence at a particular place and time, he may then prove the negative by proving a positive which implies it, as in the case of an *alibi*—that he was somewhere else at the time.

Some of the most objectionable forms of the fallacy of Irrelevant Conclusion are seen in the *argumentum ad*

hominem and kindred devices. The *argumentum ad hominem* is one which is made to rest not upon the merits of the case but upon the character and position of those who maintain it. If a man is accused of a crime, it is no answer to say that the prosecutor is as bad. If a change in the Law is proposed in Parliament, it is *ignoratio elenchi* to argue that the proposer is not the right man to bring it forward. Everyone who gives advice lays himself open to the retort that he who preaches ought to practice, or that those who live in glass houses ought not to throw stones; nevertheless there is no necessary connection between the character of the person giving the advice and the goodness of the advice. Kindred rhetorical devices are the *argumentum ad populum,* which consists in addressing arguments to a body of people calculated to excite their feelings and prevent them from forming a dispassionate judgment upon the matter in hand (the great weapon of rhetoricians and demagogues); the *argumentum ad ignorantiam,* which trades on the ignorance of the persons addressed; and the *argumentum ad verecundiam,* which is an appeal to veneration for authority instead of to reason. These illustrations naturally lead on to the subject of our next section.

2. Francis Bacon's Classification

We have already shown that in Observation, which means, at bottom, sense-perception, the mind is far from passive. Sense-perception rests, in part, on previous experience, as when wool " looks " soft, or ice " looks " hard and cold—when we do not touch it. This is the factor of interpretation. In such cases it is not usually noticed, because the interpretation is so rapid and spontaneous that it blends with the data of sense, and the whole coalesces into one experience, which appears to be immediately given.

Thus even in what appears to be a simple act of sense-perception, we must distinguish a material and a mental factor. Let us call these respectively the *objective* and the *subjective* factors. It is evident that in acts of judgment which go beyond mere sense-perception the subjective contribution is much more extensive.

Now the subjective factor may, so to speak, "get out of hand," and play a larger part than it is entitled to. What may happen, and often does happen, is thus described in the picturesque language of Francis Bacon: "As an uneven mirror distorts the rays of objects according to its own form and figure, so the mind, when it receives impressions of objects through the senses, cannot be trusted to report them truly, but in forming its notions *mixes up its own nature with the nature of things*." The result is that false images, which Bacon calls "idols" (phantoms, εἴδωλα), representations of the mind itself, come between it and reality, and vitiate knowledge. Bacon's account of these is of interest in itself and has a certain historic importance. "The foundation of axioms and theories on true Induction," he says, "is the only fitting remedy by which we can ward off and expel these idols. It is, however, of great service to point them out." Accordingly he distinguishes four classes of them.

In the first place, there are the "Idols of the Tribe" (*Idola Tribus*), which "have their foundation in human nature itself, in the tribe or race of men": as for example, the tendency to believe that things are as we wish them to be; or, again, the tendency to observe the instances favourable to any opinion which we have adopted and to ignore those which are unfavourable to it, a tendency which explains the hold of superstitions upon the human mind, and which explains also the hasty and unwarranted

inductions of "philosophers"—as Bacon calls them—who base their theories on a few outstanding facts of observation and ignore the rest.

Secondly, there are the "Idols of the Cave" or "Den" (*Idola Specus*), which "arise from the particular constitution, mental or bodily, of each individual, or from his education or habits, or from mere accident"; for "every individual has his own cave or den which intercepts and corrupts the light of nature," and thus produces errors over and above those which are common to mankind. For example, "some minds are stronger and more ready to mark the differences of things, others to mark their resemblances"; some are dominated by the love of antiquity, others by the love of novelty; and the bias of a special science or speculation affects its devotees. "Men search for knowledge in smaller worlds, and not in the greater or common world."

Thirdly, there are the "Idols of the Market-place" (*Idola Fori*), so called because they arise from the intercourse and association of men with each other. Under this head Bacon speaks specially of the misuse of *words*, and in particular of *names*, many of which become current coin without any clear or distinct ideas behind them (compare Chapter III., page 41). In spite of the efforts of thinkers to explain and define, we find that names which are mere words "force the understanding, throw everything into confusion, and lead mankind into innumerable fallacies and vain controversies." Here we may justly ask whether the men of the Middle Ages, whose methods Bacon had in his mind, were very different from ourselves. The same pathetic faith in the power of words is to be found among us as among them. "Who can wonder at past belief in the magic power of words, who

has heard statesmen speak and multitudes shout for (or against) 'Militarism,' 'Unconditional Surrender,' 'Nationality,' 'Democracy,' 'Prohibition,' 'Socialism,' 'Bolshevism'? What fears, what hopes, what passions, what prejudices, what sacrifices, these words elicit—and how little agreement there is as to their meaning!"

Finally Bacon distinguishes the "Idols of the Theatre" (*Idola Theatri*), which are "received into the mind from philosophical systems, and from perverted rules of demonstration." "These," he adds, "I call 'Idols of the Theatre,' because in my judgment all the received systems are so many stage-plays, representing worlds of their own creation, after an unreal and scenic fashion." They include not only wrong systems, but also received principles and assumptions in the sciences, belief in which has become inveterate through mere tradition and lack of criticism. In this connection Bacon refers to Plato, Aristotle, and other Greek thinkers of whom he really knew very little. We shall not inquire whether any systems of philosophy, ancient or modern, deserve the scathing description which he has given. What Bacon really means to bring out, is the danger of what he calls "Anticipation of Nature." Modern Psychology and the Modern Logic of Scientific Method agree in the verdict that *the true observer brings to his observation more than he finds in it;* and Bacon, as we have seen (page 217), was blind to the significance of this. But his real protest was against prejudices and prepossessions, induced by doctrines of various kinds, which we bring with us to the facts and which we allow to mould our interpretation of the facts. We tend to approach them with our minds already made up, in reference to their nature and meaning—with a theory "in advance," based on some "authority." Thus,

in Bacon's own time the Copernican theory of the solar system—which he himself did not accept—was struggling against the geocentric theory upheld by ecclesiastics, scholars, and most astronomers; and, much nearer to our own time, Huxley and other supporters of Charles Darwin had to fight against the dogma of "fixity of species," which was then generally accepted in science, philosophy, and theology.

3. Fallacies Deductive and Inductive

Some logicians have proposed a classification of fallacies into (*a*) those incident to deductive reasoning and (*b*) those incident to inductive reasoning. Now deductive reasoning, in the widest sense, includes Definition, Division, Immediate Inference, and Syllogistic Inference (Categorical, Hypothetical, and Disjunctive). A deductive fallacy is therefore a breach of the rules of one or other of these processes of deductive thinking; and the nature of such fallacies is naturally explained in immediate connection with the rules in question. This we have already done; and, for convenience, we now give a summary list of "deductive fallacies."

(1) Faulty Definition, including violations of any of the rules of logical Definition (see pages 43-46).

(2) Faulty Division, including violations of any of the rules of logical Division (pages 49, 50). Those which frequently occur are (*a*) change of *fundamentum divisionis*, (*b*) non-exhaustive division, (*c*) omission of steps in division.

(3) Fallacious Immediate Inference, including fallacies due to erroneous Opposition and erroneous Eduction: conversion (page 89), obversion (page 91), contraposition (page 94), inversion (page 95).

(4) Fallacious Syllogistic Inference, including (*a*) violation of the rules of the categorical syllogism, especially "four terms," "ambiguous middle," and "illicit process" (pages 127-8);

(*b*) Violation of the rules of the hypothetical syllogism (page 161);

(*c*) Violation of the rules of the disjunctive syllogism (page 165).

The Aristotelian classification cannot be satisfactorily adapted to this division of fallacies into "inductive" and "deductive" Aristotle's fallacies of "Equivocation," "Figure of Speech," "*A dicto secundum quid*" (and converse), "Composition" (and converse) may be classed as fallacies of Definition, but when expressed in syllogistic form they produce "Four Terms" "Amphiboly" and "Accent" are errors due to the misinterpretation of given statements "Many Questions" and "*Non causa pro causa*" are forms of "*Petitio Principii*"; and some forms of "*Petitio Principii*" and of "*Ignoratio Elenchi*" may be considered to be "deductive" and other forms to be "inductive" fallacies.

The Aristotelian account of logical fallacies provides no adequate descriptive analysis even of the more important errors which may occur in Inductive Reasoning. Any complete classification of "Inductive Fallacies" is impossible; but there are two broad types of fallacy in inductive reasoning, which may be conveniently distinguished under the following heads:—

(1) Fallacies of Observation.

(2) Fallacies of Generalisation (including False Analogy).

(1) We have seen that all observation and all sense-perception involve a mental factor, which may be described as one of interpretation, or more or less conscious inference. Now all inductive reasoning starts from concrete events as the basis of every hypothesis, and returns to concrete events for verification; hence its validity depends directly

on the accuracy of the observations on which it is based. Scientific observation, therefore, is a process of difficulty and danger, in which it is very easy to go wrong. John Stuart Mill distinguished two types of fallacious observation, which he called "Non-observation" and "Mal-observation": "It is non-observation when the error consists in overlooking or neglecting facts or particulars which ought to have been observed; it is mal-observation when something is not simply unseen, but seen wrongly—when the fact or phenomenon, instead of being recognised for what it is, is mistaken for something else." The distinction is convenient; but it is of psychological rather than logical importance.[1]

(*a*) We may overlook or neglect particular facts or operative conditions really pertinent to the matter which is being studied. Failure to observe such facts is usually due to one or other of the "idols" described by Bacon. We have a natural tendency to dwell upon facts which support some favourite theory, and to treat as of no importance facts which go against it, or even to neglect them altogether. Darwin thus describes a rule which he made for himself and invariably followed: "Whenever a published fact, a new observation, or a thought came across me which was opposed to my general results, to make a memorandum of it without fail and at once; for *I had found by experience that such facts and thoughts were far more apt to escape from the memory than favourable ones.*" In like manner, in the case of an accepted theory, he never let an *exception* pass. It was the exception, sometimes a small "residual phenomenon," which he seized upon, and from which he made a new start.

[1] Mill, *System of Logic*, Bk. V., Ch. v., § 1.

With the majority of men, the effects of past experience or training, of personal interest, and of prejudice or bias, are not so strenuously resisted. A man's mental habits, the surroundings in which he has been brought up, the controlling customs and ideas of his group, are powerful forces affecting what he observes and fails to observe. His training along certain lines of action renders him more sensitive to certain impressions than to others. His personal interests may render him insensible to facts which do not harmonise with them. The ideal of observation in "the dry light of the understanding" is attained, if at all, with the greatest difficulty. It is more nearly reached in such sciences as Physics and Chemistry, and is increasingly difficult of attainment as we move through the biological to the social sciences. The facts of astronomical Physics do not now affect any of the interests which tend to bias men's judgment; but it was not always so, for astronomical Physics did once interfere with some of the most deep-seated prejudices and interests of mankind.

The most extreme form of the "fallacy of non-observation" is to take for granted that because something has never been observed, it does not exist. There are indeed some cases in which such an inference is justifiable: for example, the earth's surface has been sufficiently explored as to render it highly improbable that any animal of the size of a camel remains to be discovered. To some extent it is a question of size, and of the probability (or the reverse) that such a creature would have escaped notice if it existed. If the question were asked in reference to the depths of the ocean, our answer would be less confident. The smaller the phenomenon, the more likely it is to escape observation. A good example of this is seen in the time during which the inert gas "argon," mingled in minute quantities with the nitrogen of the atmosphere,

remained undiscovered; the quantity of it is small, and it is (chemically) very inactive. The logical conclusion suggested is this: when a proposition is accepted on *negative evidence*, that is, on the ground that no case to the contrary has been observed, the most careful examination is required of the field in which such cases would occur.

(*b*) Since observation includes not only sense-perception but the interpretation of sense-perception, we must carefully guard against confusing these two factors. When this confusion occurs, we have the "fallacy of mal-observation"—that is, of wrongly interpreting what we immediately perceive through our senses. In mal-observation it is not the senses that are wrong. Thus, if we plunge our hands, one into ice-cold water and the other into hot water, and then dip both into water that is tepid, the one hand will feel a sensation of heat, the other a sensation of cold. There is no mistaking the sensations. "Mal-observation" would come in if we interpreted these sensations by saying that the water must be warmer in the one place than in the other. The sense-impression is received just as it should be under the circumstances; the error, when there is one, emerges in the interpretation of the sense-impression or the meaning which the mind gives to it. One of the most striking examples of mal-observation was the universal acceptance of the Ptolemaic solar system for many centuries. "People fancied that they *saw* the sun rise and set, and the stars revolve in circles round the pole. We now know that they saw no such thing; what they really saw was a set of appearances equally reconcilable with the theory they held and with a totally different one."[1]

[1] Mill, *Logic*, IV., v., § 5.

(2) In addition to "fallacies of observation," inductive inquiry is liable to errors due to "false analogy" and "false generalisation." These are closely akin.

We have seen (page 216) that there is an "analogy" between two objects A and B when they resemble one another in some essential or fundamental quality P, such that when we find PR in A we may expect to find PR in B. The "argument from analogy" consists in the inference *from* the presence of R in A *to* its presence in B, because P is present in both. An analogy is therefore a resemblance of relations—we have a relation PR arising from essential facts in A, and a similar relation PR arising from similar essential facts in B. We have also seen that the conclusion of an argument from analogy can only be (more or less) probable; and that the true function of such an argument is to suggest hypotheses and lines of inquiry.

The term "false analogy" is an abbreviation of the term "erroneous argument from analogy." It consists in making a false step from a relation in the case A to one in the case B. This arises when the resemblance between the two cases is not fundamental enough to bear the inference based upon it; or when, in addition to the important resemblances, there are important differences, which render any argument based upon the former merely doubtful. Examples of "false analogy" have already been given; and the reader will have no difficulty in finding others. Among "stock examples" of "false analogy" in the textbooks we often find, for instance, the "analogy" between the relation of colonies to the "mother-country" and that of children to their mother; the "analogy" between the pilot of a vessel and the ruler of a political State; the "analogy" between a national community and an individual (in reference to youth, maturity, old age, and death).

"False analogy" is clearly akin to "false generalisation." The latter is a fallacious attempt to do for many cases what the former attempts to do for two. It is a general statement made without sufficient evidence—an attempt to pass from "*This* S is P" (or *These* S's are P) to "*Every* S is P" without examining the character of the connection between S and P, which may be due to something accidental or peculiar to the special case or cases. No fallacy is more common than this. The human mind is naturally disposed to generalise its experiences, and this often means rushing into generalisations without any basis in the facts. "Nothing is more common," says De Morgan, "than to hear and read assertions made in all the form, and intended to have all the power, of universals, of which nothing can be said except that most of the cases are true. If a contradiction is asserted and proved by an instance, the answer is, 'Oh, that is an *extreme case*.' But the assertion was made of *all* cases. It turns out that it was meant only for ordinary cases. Why it was not so stated, must be referred to one of three causes: a mind which wants the habit of precision which Logic has a tendency to foster; a desire to give more strength to a conclusion than honestly belongs to it; or a fallacy intended to have its chance of reception."

In this connection we may comment on the maxim "the exception proves the rule" (*exceptio probat regulam*). If taken strictly in its literal meaning, this statement would be absurd, for it would mean that a universal proposition is "proved" by establishing the proposition which *disproves* it—namely, its contradictory. No proposition against which a single exception can be brought is really universal; and to assert it in a universal form is the fundamental essence of the fallacy of generalisation. The

statement quoted is itself a *cautionary rule*. It reminds us that an exception to a rule requires critical examination. It may then turn out to be merely an apparent exception; and in that case it may require a modification in our statement of the rule and lead to a better understanding of its fundamental meaning and range; or it may turn out to be, not a mere negative instance under the rule which we have before us, but a positive instance of *another rule* crossing the former.

It must be clearly understood that Logic can give no special rules for the avoidance of the various kinds of error which we have described in the foregoing pages. It is sufficient to point out the dangers, and to indicate the qualities of mind by which they may be avoided. These are aptly summed up by Jevons, thus: "The successful investigator must combine diverse qualities; he must have clear ideas of the results which he expects and confidence in the truth of his theories; and yet he must have that candour and flexibility of mind which enable him to accept unfavourable results and abandon mistaken views."

4. The Utility of Logic

On the subject of the practical value of deductive and inductive Logic, Macaulay gave effective expression to an opinion which has commended itself to many both before and since his time (he intended his observations to apply to both branches of Logic): "The inductive method has been practised ever since the beginning of the world, by every human being. It is constantly practised by the most ignorant rustic, by the most thoughtless schoolboy, by the very child at the breast. That method leads the rustic to the conclusion that if he sows barley, he will not reap

wheat. By that method a schoolboy learns that a cloudy day is the best for catching trout. The very infant, we imagine, is led by induction to expect milk from his mother or nurse and none from his father. . . . We conceive that the inductive process, like many other processes, is *not likely to be better performed because men know how they perform it*. . . . A man of sense syllogises in *Celarent* and *Cesare* all day long without suspecting it; and though he may not know what an *Ignoratio Elenchi* is, he has no difficulty in exposing it whenever he falls in with it." Taking "rhetoric" as an illustration, Macaulay adds that "the old systems of rhetoric were never regarded by the most experienced and discerning judges as of any use for the purpose of forming an orator"; and he quotes Cicero's statement that the power of eloquence is not achieved by rules of rhetoric, but the rules of rhetoric are based on natural eloquence.[1] This, of course, is true. But the context shows Cicero's meaning to be that the rules of "rhetoric," so far as there are such rules, have their value; that men get them by observation, and put them in practice.

Evidently the question is one of the mutual relation of theory and practice. De Morgan, criticising Macaulay, takes the illustration of tools: "The construction of tools, mental or material, is a 'cyclical' process. The first iron was obtained by the help of wood; one of the first uses of it was to make better tools, to get more iron, with which better tools still were made, and so on. And in this way we trace back any 'art' to *natural tools* and to materials which are to be had for the gathering." The rules of Logic

[1] The passage, from Macaulay's *Essay* on "Lord Bacon," is quoted in full and discussed by De Morgan, *Formal Logic*, Ch. XI. The italics are ours.

are *general results arrived at by examination of actual processes of valid thinking*. They have been gradually discovered, by starting with naturally valid thinking (practice), analysing the principles to which it conforms (theory), applying these (practice), again analysing the more adequate method of reasoning thus obtained (theory), and so on. We affirm that De Morgan's view of the subject is the true one.

Thinking and reasoning, valid and invalid, come before Logic; and though, as we have observed, the varieties of *invalid* reasoning are innumerable, there is a general resemblance between the types of *valid* reasoning in all subjects. Most thoughtful persons endeavour to form for themselves some idea of a rational method of thinking and arguing; and Logic carries this out systematically. The rules and principles of Logic are arrived at from examination of valid processes of reasoning. This is a theoretical and a systematic study. Logic is therefore rightly described as a *science*. It is true that one who has never learnt Logic may be able to arrive at valid conclusions; and the share of ordinary understanding, which men enjoy as reasonable beings, generally proves sufficient to conduct them in the ordinary affairs of life. The mistake made by Macaulay is this. He entirely overlooked the fact that the logical principles and rules, derived from analysis of valid reasoning in various typical cases, may be so stated that they can be *applied* in examining the validity of reasoning in other cases.

Logic, it has been truly said, does not make false reasoning impossible, but it does furnish rules and principles by which error can be detected and, therefore, avoided: "By bringing an argument to the test of Logic we can ascertain whether or not reason has, in that instance, been employed rightly; and, if not, what was the cause and

origin of the error. This is by no means unimportant; a man may feel sure that a conclusion is false, and yet be unable to say *why* it is so. This is frequently the case with men of 'sound common sense,' who often see at once that an argument is invalid, but are unable to point out where the fallacy lies; in which case there is of course no guarantee against committing the same error in reasoning again, when, perhaps, the conclusion may not be so obviously at fault," and the error, for that very reason, may be all the more serious.

Logic is therefore an indirect, not a direct, aid to correct thinking. It does not profess to teach us how to reason correctly in the innumerable questions which arise in science, history, and practical life; but, we repeat, it furnishes the principles and rules to which correct reasoning conforms; and by right appeal to these rules errors can be detected and avoided. It thus helps us to correct our own mistakes and to avoid being misled by others. And, in a more general way, it quickens our " sense " of bad reasoning both in ourselves and others.

We have said that Logic is a science. A " science " is in all cases a systematic or organised body of knowledge relating to some particular subject-matter; " systematic " or " organised," because knowledge of isolated facts is not science—it can only become science when such isolated facts are brought under general laws forming part of a consistent whole. The subject-matter of each science is some definite part of the material of human experience; of Arithmetic, it is the relations and properties of numbers; of Botany, vegetable life, and of Logic, thought and, in particular, thought in the form of reasoning. In some of the older books, Logic is described as an " Art "; but an " Art " is in all cases essentially practical. It is systematic *action*, which involves skill acquired by practice. It usually involves also the *application* of knowledge derived from one or more sciences, as in the " Art of Music," the " Art of Navigation," the " Art of Medicine." Logic, therefore, is not an " Art "; it is a science, capable of practical application, but it is not the business of Logic to make these applications.

5. Logic and Language

We have had occasion (see pages 17, 18) to point out the mutual *dependence of thought and language on each other*. We must now consider the most important ways in which language helps to form the logical structure of thought. This will enable us to give a definite answer to the question of how far Logic, the study of the principles of valid thinking, is concerned with language.

The influence of a developed language on the growing mind of the child is immeasurable, as he enters into it and assimilates it as part of his social inheritance.

(1) It provides him with an outline map of certain ways of thinking about the world, which are so fundamental that they underlie all other thinking. They are in fact fundamental assumptions about the nature of the world. Take, for example, two languages so different as English, representing the "Indo-European" group, and Hebrew, representing the "Semitic" group. Both of these have "nouns" and "adjectives," including adjectives of quality and quantity and demonstrative adjectives ("this," "that," etc.). These grammatical distinctions are the expression of certain "fundamental ways of thinking about the world" which arise from our experience of the world as consisting of many particular objects or individuals—"persons" and "things" distinguishable from one another by many various kinds of quality and quantity. Both languages, again, have "prepositions," distinguishing relations between things, especially relations of time, space, and causation. Both have "verbs" and "adverbs," distinguishing activities of persons and things, and qualities of their activities, together with verbal "moods," expressive of assertion, supposition, wish, and command. To the grammatical structure of language corresponds what we

must call *the structure of our thinking*: and this is of logical as well as of psychological importance.

The words and sentences that fall upon the ears of a child and are soon upon his lips, express not so much his own personal thinking as the common thinking of his kind, which becomes, as it were, a rule or measure to which his own must conform. "Why, for example, does a child have no difficulty about the relation of 'substance' and 'qualities,' which has given philosophers so much trouble? And why do all children understand it, or seem to understand it, whatever their experience may have been? Why, but because the language put into their mouths, and which they must use, settles the point for them, one and all; involving as it does a metaphysical theory which, whether in itself unexceptionable or not, has been found serviceable through all the generations of men." We use our own private experiences "mainly to decipher and verify the ready-made scheme of knowledge which is given us *en bloc* with the words of our mother-tongue."[1]

(2) None the less, within the limits of what we have metaphorically described as a "ready-made scheme of knowledge," there is room for a vast variety in the ways in which language grasps the details of experience.

A typical case of a child's invention and use of a *name* is on record. The little one learnt to say *mambro* when he saw his nurse. She worked a hand-turned sewing machine, and sometimes sang as she worked it. In the street the child saw an "organ-grinder," who was singing as he turned his handle, and exclaimed "*mambro!*" The nurse caught the meaning, and the child was overjoyed. The "organ-grinder" had a monkey; the child had an india-rubber monkey toy; he called this also *mambro*.

[1] Croom Robertson, *Philosophical Remains*, pp. 68, 69.

The name was then extended to a monkey in a picture-book. The child had a toy musical-box with a handle; this also became *mambro*, the word being extended along another line of resemblance. Soon afterwards, a strolling violin-player came within the denotation of the word. Then a very swarthy, hunch-back *mambro* frightened the child; this led to the transference of the word to a terrific coalman with a bag of coals on his back. In a short time the word had become the name of a great variety of objects having nothing whatever in common to all of them, though each was like another in some striking feature. When the application became too various it was gradually abandoned, the most impressive meaning being the last to go. In this child's vocabulary, where the word *mambro* had a run of nearly two years, its last use was as an adjective signifying ugly or horrible.

In the larger history of the language of men we see similar changes and extensions of meaning going on under similar motives, but for the most part more definitely checked and controlled by surrounding social usage and experience. The histories of words in the Oxford *English Dictionary* supply endless examples. Words change with things and the aspects of things, as these change in public interest and importance. And one result of these changes is so evident that it is usually overlooked—unless it be in the interests of some special science or of Logic. In ordinary thought people are much more sure of the particular objects to which a name belongs than they are of the qualities in the objects, on account of which the name is given. Thus, when we speak of such a thing as "a dog," "an oak tree," "a mountain," "a cottage," it is more easy to point to actual instances of these things than to explain the qualities or characteristics which we have in mind when we use these names in these ways. It is true that so long as the

qualities which govern the application of names are matters of sense-perception, little practical confusion need arise. We may not be able to give a definition of "dog," but we are very rarely in doubt as to whether a given animal is a "dog" or not; although if it were a question of purchasing or giving a prize for a "pedigree" dog, the question of its *qualities* would be of practical importance. And—to put it generally—in order to arrive at consistency with ourselves and agreement with others, we must not only be able to point to the things; we must know the characteristics which the name expresses.[1]

Now language itself provides for this fuller understanding in the wealth and variety of names which it hands on. The child finds that things, which he had already made out or discovered, *already have names;* and that the use of names leads to the discovery of their qualities. From the logical point of view, the *act of naming* provides channels for, or "canalises," *thinking*; and there are two principal ways in which it does this: (*a*) by identification; (*b*) by classification.

(*a*) To give a name to an object is to give it a certain independent existence with characteristic qualities of its own, which it must have in order that we may even recognise it; and then the generalising force of the name comes into play, leading to a further extension of the thought, namely of the object as a *kind* of thing belonging to a class with those characteristics. Mr. E. Thompson-Seton's *Biography of a Grizzly* gives an incident which, though psychologically impossible for a lower animal, affords an effective illustration for our purpose. In his "cub-hood" the grizzly, exploring the edge of a stream, had his paw caught in a beaver-trap, which he dragged

[1] The reader will remember that this distinction rests upon that of Denotation and Connotation; see Chapter II., page 20.

away with him. " His little green-brown eyes glared with a mixture of pain, fright, and fury, as he tried to understand his new enemy. He lay down under the bushes, and, intent on deliberately crushing the thing, he held it down with one paw while he tightened his teeth on the other end; . . . the trap's jaws opened, and the foot was free. It was mere chance, of course, that led him to squeeze both springs at once. He did not understand it, but he did not forget it; and he got these not very clear ideas: 'There is a dreadful little enemy that hides by the water and waits for one; it has an odd smell; it bites one's paws and is too hard for one to bite; but it can be got off by hard squeezing.' " We may safely say that this is just how a bear would not think of such an experience, and could not, unless indeed he had inherited or invented a language, and carried it to a stage of development beyond that of some types of primitive men. But the illustration shows effectively how an entirely new experience, at the human level, even when not fully understood, could *with the help of language* be analysed into details familiar in themselves, which are generalised and combined in a new way with a view to future guidance.

(*b*) This brings us to the second of the two principal ways in which language " canalises " thinking. As the child learns to use his mother-tongue, he meets with a very important difference of usage corresponding to the grammatical distinction of " common nouns " (or class-names) and " proper nouns." Some names are applicable to a number of different individuals; others are applicable only to a single individual, person, place, or other thing. Young children try to use every name as if it were a " proper noun "; but the effect of the class-name is inevitably to correct this, and set going the process of generalisation.

Generalisation, in the logical and the psychological sense, begins when we think of a number of individuals as resembling one another in some distinguishable characteristic to which we devote our attention. Even the mere "feeling of familiarity" is at least implicit recognition; it means "I have had something like this before," and to *recognise* with explicit ideas, in other words, with memory, is to assimilate or classify. We may say that "common nouns" prompt or suggest logical generalisation in the sense of classification. The "proper noun," the name of an individual, person, place, or other thing also prompts generalisation, but in a different sense. Here the "feeling of familiarity" means "this is the same individual that I have previously perceived in other circumstances." The unity of an individual is different from the unity of a class.

To sum up what has been said, we observe, (1) that language provides a scheme of fundamental and general ways of thinking about the world, and (2) that the act of *naming* promotes the logical processes (*a*) of identification, as in the analysis of a complex whole; and (*b*) of generalisation.

(3) We are now in a position to observe the difference between Logic and Grammar. Grammar studies words in themselves and in their forms of combination; it studies the form and structure of language, and language is the whole of its subject-matter. It is not so in Logic. Logic is concerned with language only as an instrument of thought. It is therefore concerned with different forms of expression in language, but only so far as they embody *differences of type in the process of thinking*. Again, since Logic considers language essentially as an instrument of thinking, its aim is so to handle the instrument as to make it a help

and not a hindrance to correct thinking. This is the reason why Logic, in dealing with any judgment or process of reasoning expressed in language, claims to be allowed to state explicitly in language all that is implicitly contained in the thought.

Some logicians have emphasised the difference between the *informative* and the *emotive* use of language—between language as expressing information and as expressing feeling or emotion. Adopting this distinction, we may say that Logic is concerned essentially with the *informative* use of language. We proceed to point out what this implies.

Language is essentially a social product; and its most obvious function is not only to provide a framework for thought as we have seen, but also to be a direct means of *communicating* thought. Why do we *express* our thoughts at all, in any kind of language? Because thought forms a common ground in which different minds can meet, and which affords them a means of mutual understanding. Every statement or assertion points outwards, by means of the words in which it is expressed, to other minds, to whom, actually or in imagination, it is addressed. Hence when our assertion is expressed in the form of a logical proposition, there are three conditions which its constituent terms must fulfil:—

(*a*) Each term ought to have the same meaning for the person using it, at one time, as it has at every other time: unless the difference is intentional and is clearly stated.

(*b*) Each term ought to have a meaning for other persons beside the one making the assertion, otherwise no thought is conveyed; and it ought to have the same meaning for all who hear it, otherwise the thought conveyed is confused or misunderstood.

Now each term not only ought to be identical in meaning for each person using it in a proposition and identical in meaning for different persons using it; it ought to be the thought of the same object, whoever may think it. The term refers to something which continues to be what it is and mean what it means whether this or that person is thinking about it or not. This may be called the "objective reference" of the term.

Hence we arrive at a third condition:—

(*c*) Each term ought always to mean the same thing, that is, it ought to have the same *objective reference* whenever it is used.[1]

The conditions state a *logical ideal*, far removed from the actual facts. There are far too many words current, whose history is like that of the child's *mambro*—except that (unfortunately in many cases) they have not dropped out of use. Generalisation, inevitable and logically valuable as it is, may lead to vagueness and confusion by extending the application of a term so as to lessen its fixed meaning, especially when the application of the name is extended to objects which have some unimportant or imaginary resemblance to those to which it was first applied. The word "law" is an important example. Its original meaning was the command of some authority having power to enforce it, and this is still its signification in Theology and Politics. Such a command led to uniformity of conduct in some particular on the part of the subjects. Hence the word "law" was generalised so as to include all cases of uniformity in the succession of natural events, leading us to expect "identical results in identical

[1] The reader will observe that these conditions state the ideal meaning of what is called the "Law of Identity" (see Chapter V., page 101).

cases." Thus arose the expression "law of nature": but from this use of the word, in connection with its original use, the idea arose that a "law of nature" must mean something more than a uniformity in the succession of natural events; and out of this idea much confused controversy has arisen.

As a matter of fact, the common understanding of many general terms, and (probably) of most abstract terms, is loose and uncertain. Take such terms as "monarchy," "tyranny," "civil freedom," "freedom of contract," "culture," "education," "temperance," "generosity," "gentleman." Imagine any discussion in which any such word is involved, and consider how far the disputants have in mind different aspects of its meaning, if not altogether different meanings for it. We may illustrate from the use of the term "monarchy."

"Monarchy," in its original meaning, is applied to a form of government in which the will of one man is supreme, to make laws or break them, to appoint or dismiss officers of State and Justice, to determine peace and war, without control by statute or custom. But supreme power has never been utterly uncontrolled in reality; and the word has been extended to cover governments in which the power of the titular head is controlled in many different ways and degrees. The existence of a head, with the title of King or Emperor, is the simplest and most salient fact; and wherever this exists, the "popular" concept of a monarchy is realised. The President of the United States has more real power than the Sovereign of Great Britain; but the one government is called a Republic and the other a Monarchy. People have discussed the advantages and disadvantages of Monarchy without first deciding whether they take the word in its etymological sense of unlimited

power, or its popular sense of titular kingship, or its logical sense of power definitely limited in certain ways. We do not attempt to draw any illustrations from the ample field of contemporary controversy; the reader will experience little difficulty in finding them. "Language," said John Locke, "being the great conduit whereby men convey their discoveries, reasonings, and knowledge from one to another; he that makes an ill use of it, though he does not corrupt the fountains which are in things themselves; yet he does, as much as in him lies, break or stop the pipes whereby it is distributed to the public use and advantage of mankind."[1]

The reader who has followed our exposition so far, will, it is hoped, now understand that the study of the principles of deductive and inductive reasoning can contribute to the realisation of that logical ideal which we have defined, so that the difference between the *ought to be* of Logic and the *is* of practical life may be less vast.

[1] Locke, *Essay Concerning Human Understanding*, Bk. III., Ch. XI.

GLOSSARY

Many technical terms which are of comparatively small logical importance are current in the discussion of logical questions. We append a series of brief explanatory notes on technical terms of this kind, which the student is likely to meet with in examination papers and elsewhere, and which have not been definitely explained in the preceding pages. We include also brief notes on a number of terms, the full treatment of which lies beyond the limits of an elementary work (these are marked with an asterisk).

Accent (*accentus*), *fallacy of*. As explained in modern textbooks, this consists in perverting the original and intended meaning of a sentence by false emphasis in speaking or writing. Thus, if in quoting an author we italicise a word which he has not italicised, or leave out words in the quotation or its immediate context, we are guilty of this fallacy.

Accident (*accidens*), *fallacy of*. This consists in confusing an essential with an unessential difference or resemblance. Thus: "Is Plato different from Socrates?" "Yes." "Is Socrates a man?" "Yes." "Then Plato is different from man." It does not follow that because the one differs from the other in certain respects they differ in every respect, or that they are similar in all respects because they resemble one another in certain respects. Of the latter mistake the following is a crude example: "To call you an animal is to speak the truth, to call you an ass is to call you an animal; therefore to call you an ass is to speak the truth."

Some logicians have confused the fallacy of Accident with the fallacy *A dicto simpliciter ad dictum secundum quid* (see page 154).

Added Determinants. Immediate Inference by "added determinants" consists in adding the same "determinant" or qualification to the subject and predicate of a proposition. Provided that the qualification added to the predicate is *in all respects the same* as that added to the subject, the truth of the new proposition follows immediately from the truth of the original one. Errors may arise through overlooking the caution which we have italicised. Thus: "All kings are men," but it does not follow that "All incompetent kings are incompetent men." The fallacy is due to the fact that a "determinant" which is intended to apply to the subject-class alone, is applied in the predicate to the whole class of which the subject is only a part. The "determinant" therefore is

not the same in the two cases because its reference or application is not the same Some writers have distinguished from this what they call "Immediate Inference by Complex Conception" "Triangles are plane figures, therefore the study of triangles is the study of plane figures" This process is valid under the same precautions as in the case of "Added Determinants" The two processes have been quite properly classed together under the head "Immediate Inference by Complication of Terms."[1]

Amphiboly (*amphibolia*), *fallacy of*. This arises when the ambiguous grammatical structure of a sentence produces misunderstanding. Examples are not uncommon in the daily press and elsewhere: "The lava-stream flowed over the road to the crater made by Mussolini's orders." This misunderstanding, when it really occurs, is due to want of precision in the order in which the terms are arranged. It may also be due to poetical or rhetorical considerations: "The Duke yet lives that Henry shall depose" (*King Henry VI., Part II.*, Act. I. Sc. iv.).

Ampliative propositions See "Analytic and Synthetic propositions."

**Analytic and Synthetic propositions.* The distinction of "analytic" and "synthetic," in reference to propositions, depends on an assumed *fixity of definition* in the case of the subject. The proposition "S is P" is "analytic" (sometimes also called "explicative," and, very misleadingly, "verbal") when P is the definition or part of the definition of S; it is "synthetic" ("ampliative" or "real") when P is not part of the definition of S. It is evident that only when we have an *accepted definition* of the subject can we tell whether the proposition is "analytic" or not.

Analytic Reasoning. See "Regressive Reasoning."

Apodeictic propositions. This is another name for "necessary" propositions, of which the form is "S must be P," or "it is the nature of S to be P."

**A priori and a posteriori*. In the most general sense of the term, we are said to argue *a priori* when we argue "from the nature of the case," independently of any verification of the conclusions at which we arrive. *A priori* reasoning, in this sense, claims to anticipate the event. A proposition is said to be known *a priori* when it is axiomatic (see page 100). An *a posteriori* proposition claims to be based on observation; and *a posteriori* reasoning is equivalent to inductive reasoning.

[1] See Professor A. Wolf's *Textbook of Logic*, Ch. VII., p. 78.

Blind experiment. This is a less common name for "negative experiment," by which, after having verified the proposition "If A, then C," we seek to establish the proposition "If not A, then not C" (see page 203).

**Categories* (*Aristotelian*). Every affirmative proposition predicates some kind of being or existence of its subject, and the Aristotelian theory of the "Categories" is an attempt to classify the "kinds of existence" which can be predicated in logical propositions. Aristotle drew up a list of nine kinds or modes of existence which can be predicated of an *individual* or a *class* (these being the two forms of the first and most fundamental of the "categories," namely "substance," οὐσία). The nine other "categories" are "Quantity," "Quality," "Relation" (limited mainly to correlation), "Place," "Time," "Posture" (attitude), "Having" (possessing), "Doing," "Suffering." The list is open to much criticism: for example, the "categories" are not all equally fundamental.[1]

Categorematic word. A word which can be used as a logical Term, *i e.* as Subject or Predicate of a proposition in logical form.

Causa cognoscendi. The *reason for our knowledge* of an event, whether arrived at by inductive or by deductive reasoning, or the premisses on which the inference is based.

Causa essendi. The actual cause of an event as understood in inductive Logic.

Chain of syllogisms. A series of syllogisms bound together by a real connection or relation of logical dependence. A common form is that in which each syllogism either supports, or is supported by, the next one (see page 152).

Cognate species. A less common name for "co-ordinate species" (see below).

Cognate genus. A wider or higher class (not necessarily the "proximate" genus) under which a species is contained Thus, "plane rectilineal figure" is a cognate genus of "equilateral triangle."

Complex conception. See "Added Determinants."

Complication of terms. See "Added Determinants."

Consequent (*consequens*), *fallacy of.* By the Greek logicians this term was used of the invalid argument from the affirmation of the consequent or denial of the antecedent in a hypothetical syllogism. The most dangerous form of it is the assumption that because a conclusion is supported by invalid arguments it must be false.

[1] See Joseph, *Introduction to Logic*, Ch. III.

In some modern textbooks the fallacy of the "consequent" is explained as meaning any kind of "inconsequent" or illogical argument.

Consilience of inductions. We are said to have a "consilience of inductions" when an inductive conclusion from one class of facts (or one department of knowledge) supplies an unexpected explanation of another class of facts (in another department of knowledge).

Co-ordinate species. Constituent species of the same genus, each being marked off from it by a characteristic *differentia.*

Converse relation. Immediate inference by "Converse relation" is the name given to the process by which, *from* a statement of the relation in which P stands to Q we *pass to* a statement of the relation in which Q consequently stands to P. The terms of the original proposition are transposed, and the word by which their relation is expressed is replaced by its correlative (see page 29). "A is the father of B," "B is the child of A"; "A is older than B," "B is younger than A", and so on.

Crucial instance A test case which will decide between two rival hypotheses which have been held concerning the facts under investigation. When the test case is found by experiment, it is called a "crucial experiment" (*experimentum crucis*).

Epicheirema. A chain of abridged syllogisms in which one premiss of each syllogism is omitted. A simple form of it occurs in a syllogism in which one of the premisses (or both) is expanded by the addition of a reason: "All unnecessary duties on imports are harmful, as they impede the trade of the country; the protective duties in question are unnecessary, as they support industries which are able to stand alone; therefore the protective duties in question are harmful."

Episyllogism and Prosyllogism. In a chain of syllogisms where the conclusion of one furnishes a premiss of the next, the syllogism whose conclusion furnishes one of the premisses of the next is called the "prosyllogism," and the syllogism which uses a previous conclusion for one of its premisses is called the "episyllogism." A "Sorites" (page 152) is therefore a chain of prosyllogisms and episyllogisms in which all the conclusions (except the last) are omitted.

Experimentum crucis. See "crucial experiment."

Exponible proposition. A proposition whose logical composition is not evident from its form. "Exclusive" and "exceptive" propositions are of this kind. "Only S is P,"—"Only virtue can make a man truly happy", and "All S but XS is P" (or "No S

but XS is P"),—"Nothing but virtue can make a man truly happy" (see page 67).

Extremes of syllogism. The Subject and Predicate of the conclusion (as contrasted with the Middle Term).

Figure of Speech (*figura dictionis*). As a fallacy, this is the confusion of supposing that words similar in being derived from the same root are similar in meaning. It is a kind of false analogy. For example, J. S. Mill, in a well-known passage in his *Utilitarianism*, argues as if he supposed that the meaning of "desirable" is analogous to that of "audible," "visible," etc

Fundamental syllogism. A syllogism in which the middle term is distributed only once, and neither of the "extremes" is distributed in the premisses without being also distributed in the conclusion. Of the nineteen valid moods named in the mnemonic lines, all except *Darapti*, *Felapton*, *Fesapo*, and *Bramantip* are "fundamental" (see "Strengthened Syllogism").

Hysteron proteron (ὕστερον πρότερον). A name sometimes given to that form of *Petitio Principii* in which the fallacy is committed in a single step of inference (see page 272).

Indefinite name (*or term*). The purely formal contradictory of a term "P," covering everything in the universe which *is not* "P" (see page 27).

Indesignate proposition. A proposition to whose Subject no mark of quantity is attached. Such a proposition is not in logical form; the Subject must be "qualified" by the addition of "All," "Some," "No," according to the logical emphasis intended by the proposition

Linea predicamentalis. A "predicamental line" is the name sometimes given to a series of Terms arranged according to their decreasing denotation, beginning with the *summum genus* and ending with the *infima species* (see page 32).

Modus ponens, modus tollens. The *modus ponens* of a hypothetical syllogism is a valid inference from the affirmation of the antecedent; the *modus tollens*, from the denial of the consequent. Both the *modus ponens* and the *modus tollens* may be sub-divided into four classes. In both cases the names of the classes are the same: (*a*) *Modus ponendo ponens*, where both minor premiss and conclusion are affirmative; (*b*) *Modus tollendo tollens*, where they are both negative; (*c*) *Modus ponendo tollens*, where the minor premiss is affirmative and the conclusion negative; (*d*) *modus tollendo ponens*, where the minor premiss is negative and the conclusion affirmative.[1]

[1] Latin, *ponens*, "affirming" or "which affirms"; *tollens*, "denying" or "which denies"; *ponendo*, "by affirming"; *tollendo*, "by denying."

Thus, the last-named " mood " may be symbolised: " If not A, then C; not C; therefore A "; *or* " If not A, then C, not A; therefore C."

Nomenclature. A " nomenclature " is a system of names for the groups composing a systematic classification. Only those sciences, such as Botany and Zoology, which have a generally received classification, have a true general nomenclature.

Non causa pro causa. This expression might be used to describe the inductive fallacy of attributing an event to the wrong cause. It has also been employed in a more general sense, of the attempt to derive a conclusion from premisses which do not warrant it (for which the expression *non sequitur,* " does not follow," is more suitable). Originally it was employed in a special sense, as descriptive of a misuse of the kind of argument called *reductio ad absurdum* or *ad impossibile.* When you intend to refute a given proposition by showing that it leads to impossible or absurd conclusions, you must include that proposition in the premisses which lead to the absurdity. But it may happen that the absurdity follows from some other propositions in your premisses, and not at all from the disputed proposition. To overlook this, is the fallacy of *non causa pro causa* in the older meaning of the term.

Non sequitur, fallacy of. See *Non causa.*

**Nota notae.* An abbreviation standing for the expression *nota notae nota rei ipsius* (with its corresponding negative, *repugnans notae repugnat rei ipsius*), which is the *dictum de omni et de nullo* (see page 144) stated in terms of *connotation* instead of denotation: " Whatever is an attribute of any attribute, is an attribute of that which this latter is an attribute of "; " if A is an attribute of B and B an attribute of C, then A is an attribute of C."

Ostensive reduction. A less common name for " direct reduction " (see page 140).

Paralogism. This term has sometimes been used as synonymous with " fallacy "; but if it is used at all in Logic, it is best used as a name for mistakes in *deductive* reasoning.

Plures interrogationes. The fallacy called by this unsuitable name, " many questions," consists in demanding " a plain answer *yes* or *no*," to a question which really implies an assumption; for example, " Have you abandoned your intemperate habits yet ? "

Prosyllogism. See " episyllogism."

**Progressive reasoning.* A chain of syllogistic reasoning in which we pass from prosyllogism to episyllogism. The term may also be applied to more elaborate and complex trains of deductive reasoning

in which the conclusion established by one section of the argument becomes the starting-point for further deductions. Reasoning of this kind is also said to be "synthetic."

**Regressive reasoning*. A chain of syllogistic reasoning in which we pass from episyllogism to prosyllogism; also called "analytic" reasoning.

Repugnant names. Names which are mutually exclusive without being contradictory (negating each other) or contrary (expressing extremes of difference); for example: "red" and "green"; "woollen" and "silken."

Sophism. A fallacious argument employed with the intention of deceiving. But Logic is not concerned with the intentions of the person employing an argument.

Strengthened syllogism. A valid syllogism in which one of the premisses is stronger, or states more than enough, to establish the conclusion. Thus, among the valid moods named in the mnemonic lines, there are four "strengthened" syllogisms: in *Darapti, Felapton*, and *Fesapo* the middle term is distributed twice; and in *Bramantip*, the major term is distributed in the major premiss and not in the conclusion (see *Fundamental syllogism*). On the other hand, a "weakened" syllogism is one in which the conclusion is weaker than the premisses warrant. This occurs when a "particular" instead of a universal conclusion is deduced in *Barbara, Celarent, Cesare, Camestres*, or *Camenes*. These syllogisms are sometimes called "subaltern moods." They are not only useless but misleading, as they suggest that the universal conclusion cannot be drawn.

Subalternant (*subalternans*). Names given to a universal proposition when regarded as in "opposition" (see page 87) to its corresponding particular proposition, called the "subaltern."

Sui generis. An object is said to be *sui generis* when it is unique, in the sense that it cannot be classified in the same genus with other objects. Thus, the "rings of Saturn" are a phenomenon *sui generis* in the solar system.

Synthetic reasoning. See "Progressive reasoning."

Terminology. A scientific "terminology" is a system of names which will enable us to describe the objects studied in the science, using the terminology to identify their parts and properties. Botany affords an excellent example of an exact and extensive terminology. Every part of a plant, and the form of every part, can be identified by a technical term.

* *Vera causa* When the cause suggested in a hypothesis is a cause otherwise known to exist, and is not assumed entirely *ad hoc* for the problem before us, it is said to be a *vera causa* or "real cause." A hypothesis suggesting a "real cause" (such as the pressure of the atmosphere) is to be preferred to one suggesting a purely hypothetical cause (such as Nature's "abhorrence of a vacuum"); and among "real causes," preference is to be given to those which are known not only to exist but to be capable of acting in the way required.

QUESTIONS AND PROBLEMS FOR REVISION

(It is assumed that the student has read the whole of this book.)

CHAPTERS I. AND II

1. Explain what is meant by Inference, and mention some of its *propria* and inseparable *accidentia*.

2. "If Inference goes beyond its premisses, it is invalid; and if not, there is no advance in knowledge." Examine this statement.

3. "This may be an Inference to you; but it has long ceased to be one with me, for I worked it out long ago." Criticise the view of Inference here implied.

4. Define "Judgment" and "Proposition," from the point of view of Logic.

5. Can a *logical* distinction be drawn (*a*) between noun and adjective, (*b*) between preposition, conjunction, and adverb?

6. Which of the usual divisions of Terms or Names do you consider to be of fundamental importance in Logic, and why?

7. What is meant by the Connotation of a name? Does the Connotation of a name include *all* the qualities possessed by the objects to which it applies, or *only some* of them? Give reasons for your answer.

8. "As the Connotation of a name is increased, its denotation is decreased." Discuss this statement, with examples. Are there any names whose Connotation may increase without any consequent increase in their Denotation, and *vice versa*?

9. Give examples showing different ways in which the Connotation of a name may be limited so that the name is applicable only to one object. Distinguish such names from Proper Names.

10. "Has a Proper Name *meaning*?" "Has a Proper Name *connotation*?" Discuss these questions, carefully explaining what you understand by "meaning" and by "connotation."

11. Can any definite and logically useful meaning be attached to the word *connotative*, used as an adjective? If so, are any names "non-connotative"?

12 "Not every name denotes, nor does every name connote" Explain the meaning of this statement, and give your reasons for agreeing or disagreeing with it.

13. Consider which, if any, of the following names have (*a*) no connotation, (*b*) no denotation; giving your reasons:—"Politician"; "The Crown Prince of Ireland"; "New York"; "Dragon"; "Ghost."

14. Explain clearly the distinction between Singular and General names, and illustrate your answer by a discussion of the logical character of each of the following:—"The strongest man on earth"; "The Rt. Hon. D. Lloyd George"; "The University of London"; "Logic"; "Science"; "Iron"; "Sound"; "City."

15. Compare (*a*) the distinction between the Concrete and Abstract use of names as Terms, with (*b*) the distinction between nouns and adjectives May differences of *quantity* be admitted in the case of Abstract Terms? How many different kinds of Concrete Terms may be recognised?

16. Explain and illustrate the difference between a *collective* and a *general name* (class-name).

17. What are the *Predicables?* Write down *five* propositions, all of which have the term "triangle" for subject, but in which (in each case) a different Predicable is asserted of the subject.

CHAPTER III

18. What purpose is served by Definition? Explain clearly exactly why "circular definition" is useless.

19. Carefully explain definition *per genus et differentiam*. What kinds of names are indefinable, and why?

20. In what sense may we speak of different "kinds" or "types" of Definition? Give examples.

21. Explain, with illustrations, the principal faults which may occur in Definition.

22. Critically examine the following Definitions:—(*a*) "Faith is a vital working interest in anything"; (*b*) "Wealth is that portion of the produce of land and labour which is capable of being accumulated"; (*c*) "Logic is a good mental discipline"; (*d*) "Ignorance is a blind guide"; (*e*) "A straight line is that which lies evenly between its extreme points" (Euclid gives this definition after stating that "a line is length without breadth" and that "the extremities of a line are points").

23. What constitutes a good Classification? How is Classification related to logical Division?

24. Give an example of Division by Dichotomy. Why is Dichotomy not a successful scientific method? In what cases is Division by Dichotomy the only method possible?

25. Give examples (*a*) of classifications which may be called "natural," and (*b*) of classifications which may be called "artificial" Explain the distinction between the two kinds.

26. Explain and illustrate the meaning of the terms *Fundamentum Divisionis* and "Cross Division."

27. Examine critically the following Divisions:—(1) Pictures into paintings, posters, photographs, and pencil drawings; (2) Government functions into legislative, executive, and judicial; (3) Political parties into Socialists and Conservatives; (4) Books into historical, scientific, religious, and political, (5) Plants into flowering and non-flowering; (6) Triangles into equilateral (three equal sides), isosceles (two equal sides), and scalene (three unequal sides); (7) Poems into epic, lyric, and dramatic; (8) England into forty counties; (9) Mind into "thought," "feeling," and "will"; (10) Terms into singular, general, abstract, and concrete.

CHAPTER IV

28. What is meant by "Quantity" and by "Quality" in the traditional classification of logical propositions? Show how this classification naturally follows from the marks of "Quantity" and "Quality" which have been admitted.

29. Discuss typical examples of the marks of Quantity used in statements occurring in ordinary speech, *other than* the traditional logical and the definitely numerical marks.

30. If "some" were used in the sense of "some only" (*i.e.* "some but not all"), what would be the exact meaning of an I proposition? How would it be related to O? And what information would the denial of I convey?

31. What is the difference between the assertion made in a proposition of the "subject-attribute" type and the assertion made in one of the "relational" type? Give five significant (non-trivial) examples of each type.

32.* Carefully distinguish the logical nature or structure of each of the following propositions:—(*a*) "Socrates is a man"; (*b*) "All men are fallible"; (*c*) "If he is human he is fallible"; (*d*) "The Athenian democracy put Socrates to death."[1]

[1] Questions 32-34 (inclusive) relate directly to Chapter IV., § 4.

33 * Critically examine the logical classification of propositions into the four types represented respectively by A, E, I, and O. Outline a more adequate classification.

34.* Discuss the logical nature or structure of each of the following propositions:—

(1) " So careful of the type she (Nature) seems."
(2) If you do that, I will punish you.
(3) If a man holds an important post, he is not always competent.
(4) There are few minds but might furnish some instruction
(5) Vice never brings happiness
(6) The unknowable is the unthinkable.
(7) All the plays of Shakespeare cannot be read in a day.
(8) Every inhabitant of London is either British or Foreign.
(9) This hat is either yours or mine.
(10) Lying is the result either of vanity or cowardice.

35. Explain the logical ambiguity which may occur in a proposition expressed symbolically in the form " S is either P or Q."

36. Express each of the following propositions in logical form (*i.e.* in one or other of the four forms A, E, I, or O), giving the correct symbol in each case.—

(1) Anyone can do this.
(2) None were saved.
(3) All politicians are not dishonest.
(4) Philosophy bakes no bread
(5) Only those who are unselfish are happy.
(6) The dead have no rights.
(7) Critics are unsuccessful authors.
(8) A bird in the hand is worth two in the bush.
(9) Self-pity is not incompatible with arrogance.
(10) A single experience sometimes changes our whole outlook on life.
(11) Noble things are as difficult as they are rare.
(12) There are many such in the country.
(13) The good can only be preserved by transforming it into the better.
(14) None came to grief except those who paid no heed to the warning.

(15) No persons will be admitted but by special permission.

(16) There are frequent contradictions in the daily newspapers.

(17) That is a mistake.

(18) To forgive is divine.

(19) Nothing was a trouble to him.

(20) Among Englishmen many great poets are found.

37. Explain the relation between categorical, hypothetical, and disjunctive propositions. Is it possible to express the same truth in all three forms?

CHAPTER V

38. Define Immediate Inference, showing (*a*) in what sense it is immediate, (*b*) in what sense it is Inference.

39. Explain and illustrate the following.—" Of propositions which are logically ' opposite,' those related by *subalternation* may both be true and may both be false, those related as *contradictories* may not both be true and may not both be false; those related as *contraries* may not both be true and may both be false; those related as *subcontraries* may both be true and may not both be false."

40. Carefully distinguish the use of the words " Contrary " and " Contradictory " as applied to *propositions* from their use as applied to *terms*. Illustrate your answer by reference to the terms " useful," " useless," " successful," " unsuccessful," and to propositions in which these terms occur as predicates.

41 Given the three classes " rebels," " patriots," and " brave (men)," show what relations must hold among them if the following propositions are true: " No rebels are patriots "; " Some rebels are brave "; " Some patriots are brave."

42. What propositions must be true and what false, (*a*) if it is *true* that " There is some good in every man "; (*b*) if it is *false* that " a whale is a fish "; (*c*) if it is *true* that " comparisons are odious " ? (Only the relations contained in the " Square of Opposition " need be considered)

43. (*a*) Given that an I proposition is *true*, what may be inferred as to the truth (or otherwise) of the A and E propositions having the same subject and predicate? (*b*) Given that an I proposition is *false*, explain why an O proposition with the same subject and predicate cannot also be false.

44. State in logical form the propositions which are respectively *necessary and sufficient* to *deny* each of the following:—

(1) The necessaries of life should be obtainable by all with a reasonable amount of effort.
(2) Many who do not work hard rise to positions of eminence.
(3) There is nothing new under the sun.
(4) "Never yet was noble man but made (*i.e.* became the subject of) ignoble talk."
(5) I alone have found the truth.
(6) The reward of one duty is the power to fulfil another.
(7) Natural productions are not all equally perfect.
(8) Health cannot be long maintained without exercise.
(9) "Some mute inglorious Milton here may rest."
(10) There is one thing only which gathers people into seditious assemblies, and that is oppression.

45. Discuss the logical value of denying a proposition by its Contrary; and give the Contrary of each universal proposition in the previous question (44).

46. Name and define the different kinds of Immediate Inference which are possible by "Eduction."

47. Give the Converse of the Contrary of the Contradictory of the proposition, "Some men are learned." How is it related to the original proposition?

48. What types of proposition (*a*) cannot be converted; (*b*) cannot be converted simply? Explain the reasons carefully.

What is the logical relation between "All who practice medicine are not qualified physicians," and "Only qualified physicians practice medicine"?

Give the obverted converse of the obverted contrapositive of the second proposition. What is its logical name?

49. Point out the logical relation (if any) which each of the following propositions holds to the proposition "All dealers are producers":—

(1) None who are not producers are dealers.
(2) Some producers are not other than dealers.
(3) Some dealers are persons other than producers.
(4) Some dealers are not persons other than producers.
(5) Some who are not dealers are not producers

(In this question, "logical relation" may be taken to mean any of the relations given in the Square of Opposition or the Table of Eductions.)

50. Given that the proposition "No unambitious man is successful" is true, state what is known as to the truth or falsity of each of the following propositions, naming its logical relation to the given proposition:—

(1) Ambitious men are sometimes successful.
(2) Only unsuccessful men are unambitious.
(3) Not a few successful men are unambitious.
(4) Some ambitious men are not unsuccessful

51. Given a true proposition of the form "None of the righteous pitied him," point out what can be inferred as to (*a*) "the unrighteous," (*b*) "those who did not pity him."

52. State each of the following propositions in logical form, and give (where possible) the Converse, the Obverse, and the Contrapositive of each:—

(1) Uneasy lies the head that wears a crown.
(2) Not all schemes of social reform are feasible.
(3) Only unscientific people are credulous.
(4) Great happiness is sometimes too profound to pass away.
(5) No man can ever be other than himself.
(6) Pure gold is the most malleable of all metals.
(7) The longest road comes to an end.
(8) All are not happy that seem so.
(9) It cannot be that none will fail.
(10) No mathematician believes that he has squared the circle.

53. Discuss the application (*a*) of logical Conversion, (*b*) of logical Contradiction, to propositions whose subjects are Singular terms, taking the following as examples:—

(1) Socrates was the wisest man in Athens at that time.
(2) "Great is Diana of the Ephesians."
(3) The sun is a very small star.
(4) That is exactly what I wanted.
(5) He went to London.

54. What do you understand by "Law," in the expressions "a Law of Thought," "a Law of Nature," "Criminal Law"?

55. State the Laws of Contradiction and of Excluded Middle so as to bring out the essential logical meaning of each. Are they mutually independent? Give your reasons.

56. For the purposes of Logic, what serviceable meaning, or meanings, may be given to the " Law of Identity " ?

57. In what sense (if any) can (*a*) a Law of Nature, (*b*) a Law of Thought, be " violated " ?

CHAPTER VI

58. Discuss the distinctive character of " Deductive Reasoning."

59. What is a Syllogism ? Explain the meaning and implications of the phrase " Middle Term."

60. State, explain, and illustrate the Rules relating (*a*) to the Distribution of Terms, (*b*) to the occurrence of Negative Premisses, in a Syllogism.

61. " At least one premiss of a valid Syllogism must be a universal proposition." Why ?

62. Prove that if one premiss is particular, the conclusion must be particular.

63. Test each of the following arguments directly by the General Rules of the Syllogism.—

(1) All men have a right to express their opinions freely; what is right should be done in all cases, therefore we should always express our opinions freely.

(2) Anything opposed to industrial prosperity is an evil; wars are certainly evils; therefore wars are opposed to industrial prosperity.

(3) All patrons of arts and science are public benefactors; no poor men are patrons of art and science; therefore no poor men are public benefactors.

(4) All men are rational beings; all rational beings are progressive beings; therefore all progressive beings are men.

(5) No conservative government ever permits a revolution; all governments permitting a revolution are unsuccessful; therefore all conservative governments are successful.

(6) Some leaders of industry are advocates of a high tariff; some advocates of a high tariff are members of Parliament; therefore some members of Parliament are leaders of industry.

64. Prove from the General Rules (*a*) that if the conclusion of a valid Syllogism is universal, the Middle Term can be distributed only once; and (*b*) that if the Middle Term of a valid Syllogism is twice distributed, the conclusion cannot be universal.

65. Explain the expressions "Major Term," "Minor Term," "Major Premiss," "Minor Premiss," "Figure," and "Mood," as applied to Syllogism. Account for this special use of the words "Major" and "Minor."

66. Prove from the General Rules that in a valid Syllogism no conclusion can be obtained from a particular major and a universal negative minor premiss. Prove also that if these premisses be transposed a valid conclusion can be obtained in all Figures

67. Give an example (in words) of a Syllogism in the first Figure with a particular negative conclusion. Using the contradictory of this conclusion as a minor premiss, combine it with the original negative premiss to make another Syllogism. Name the Figure and Mood of the new Syllogism, and point out the relation of its conclusion to the original affirmative premiss.

68. Briefly discuss the characteristics of each of the first three Figures. Is there any reason for asserting the superiority of the first Figure?

69. Show, from the nature of the reasoning expressed in the first Figure, that its major premiss must be universal and its minor premiss affirmative. Prove this also from the General Rules.

70. The conclusion of a valid Syllogism is a universal affirmative. Determine the Figure and Mood (directly from the General Rules).

71. "The so-called 'Special Rules' of the syllogistic Figures are simply applications of the General Rules to syllogisms of these particular forms respectively." Illustrate this in the case of the second, third, and fourth Figures.

72. Discuss the logical significance and application of the *Dictum de omni et de nullo*.

73. Give the complete form of the following abridged syllogism, and express it in as many valid Moods as possible. "Some theorists cannot be trusted, for they are inexperienced."

74. What is "Reduction"? Why did the Greek logicians consider it necessary?

Explain the process called "Direct Reduction," taking as examples the following syllogisms, each of which is to be "reduced directly":—

(1) All dogs are intelligent, and no intelligent creatures are brainless, hence no brainless creatures are dogs.

(2) Some gratifications of appetite are injurious to health; all such gratifications are pleasant at the moment; therefore some things which are pleasant at the moment are injurious to health.

(3) The instincts are congenital endowments, and the virtues are not congenital endowments, hence the virtues are not instincts.

75 What is meant by "Indirect Reduction"? Why was it originally invented? Illustrate the process by taking as examples the following syllogisms, each of which is to be "reduced indirectly":—

(1) All acts of moral choice are of the possible; some desires are not for the possible; hence some desires are not acts of moral choice.

(2) Some persons accused of witchcraft have not believed themselves to be innocent, all accused of witchcraft are accused of a merely fictitious crime; hence some accused of a merely fictitious crime have not believed themselves to be innocent.

76. Show how each of the two syllogisms given in the previous question (70) may be reduced *directly* to the first Figure (avoid uncouth or cumbrous statements of the propositions).

77. Distinguish a Categorical from a Hypothetical Proposition. State the rules of the hypothetical syllogism, and show the connection between these and the rules of the categorical syllogism

78. "To affirm one of two alternatives is to deny the other." "To deny one of two alternatives is to affirm the other" Under what conditions are these rules respectively applicable to a Disjunctive Syllogism?

79. State the essentials of the kind of argument called "Dilemma." On what does the validity of a Dilemma depend? What is meant by "rebutting" a Dilemma?

80. Examine the following: "One of the two propositions A and B must be false; for if A is true so is C, and if B is true, so is D: but it is known that either C or D is false"

81. Explain clearly the nature of the fallacy involved (*a*) in denying the antecedent, (*b*) in affirming the consequent.

82. What is an "Enthymeme"? How would you proceed in endeavouring to decide whether the inference "No M is S, therefore some S is not P" is or is not valid?

83. "Let the Enthymeme be described as an argument in the form in which it would naturally occur in thought or speech." Critically examine this. Can a useful distinction be drawn between the natural order and the logical order in thinking?

84. "The Rules of a 'Sorites' are nothing but the special rules of the first Figure." Explain and illustrate this statement.

85. Describe the logical character of each of the following arguments; and examine the validity of each, stating it (where necessary) in full and in logical form:—

(1) Those who have no occupation have nothing to interest themselves in, and therefore are unhappy; for men with nothing in which to interest themselves are always unhappy, since happiness depends on the success with which we advance the objects in which we are interested; and so wealth is no guarantee of happiness.

(2) If the price of an imported article rises, those who manufacture the same article at home will charge more for it, if a tax is imposed on the importation of an article, the price of the imported article rises, therefore if a tax is imposed on the importation of an article, those who manufacture the same article at home will charge more for it.

(3) If the North American colonies were unrepresented in Parliament, they ought not to have been taxed by Parliament; but they were unrepresented in Parliament, therefore they ought not to have been taxed by Parliament.

(4) The belief in a Golden Age rests either on history or on hope: it does not rest on history, hence it rests on hope.

(5) Either Newton or Leibniz invented the Calculus. Newton invented it, therefore Leibniz did not.

(6) If there is censorship of the press, abuses which should be exposed will be hushed up; and if there is no censorship, truth will be sacrificed to sensation: but there must either be censorship or not, therefore abuses which should be exposed must be hushed up, or truth be sacrificed to sensation.

(7) If Homer speaks truth about things divine, the heroes were sons of gods *and* did many wicked things; but either they were not sons of gods or they did not do wicked things: therefore Homer does not speak truth about things divine. (Plato, *Republic*, Bk. III., p. 391.)

(8) Those cultivate the land best who have a personal interest in its improvement; hence peasant proprietors are the best cultivators.

(9) Every soldier serves his country; women are not soldiers, therefore no women serve their country.

(10) "You, as you are old and reverend, should be wise."[1]
(*King Lear*, Act I., Sc. ii.)

86. Discuss the claim of the Syllogism to cover all forms of Deductive Reasoning. What kind of Relation is expressed most naturally in syllogistic forms?

87. Discuss the character and form of the following arguments:—

(1) A lies to the south-east of C, being due south of B, which is due east of C.

(2) M is equal to N, for both are equal to P.

It is sometimes said that such arguments "have no Middle Term." In what sense is this true?

88. Give examples of the argument *a fortiori*.

89. The following series consists of deductive arguments of different kinds: state each argument (when necessary) in strict logical form, point out its logical character, and examine its validity:—

(1) It is not true that everything which affords satisfaction is wealth; for personal friendship is not wealth.

(2) There is a purpose in everything reasonable, and therefore all amusements, being purposeless, are irrational.

(3) This author is certainly confused. If I understand his book rightly, he is confused in his thinking, and if I do not understand it, he is confused in his writing

(4) People never learn anything about Beauty, because they start with the idea that they know all about it

(5) He speaks the truth, and a man who speaks the truth is always worth hearing

(6) He must be mad to do such a thing.

(7) Not all educated persons spell correctly, for one often finds mistakes in spelling in the papers of University students.

(8) A system of Free Imports is a benefit to the wage-earning classes, for it increases trade, and thus cheapens articles of ordinary consumption; this gives a greater purchasing power to money, which is equivalent to a rise in real wages and so is a benefit to the wage-earning classes

[1] For this series of examples I am indebted to Joseph, *Introduction to Logic*, 2nd ed., Ch. XV., XVI.

(9) The envious are never happy, for they are never content with what they have.

(10) Suicide is not always to be condemned; for it is but voluntary death, and voluntary death has been gladly embraced by many heroes.

(11) M and N have equal incomes, but it does not follow that they are equally wealthy.

(12) If the train is late, I shall miss my appointment; if it is not late, I shall miss it (the train); therefore in any case I shall miss my appointment.

(13) Mr. S. is a solicitor, and the worse his case the better he argues; and he has argued very well to-night.

(14) The theory of Evolution is true, because it is accepted by every competent biologist.

(15) It is unjustifiable to attempt to relieve unemployment by legislative measures. If trade is recovering there need be no such measures, and if it is not, none are of any avail.

(16) Knowledge is desirable because power is desirable, and knowledge gives power.

(17) Comedy is possible only in a highly civilised country, for in a comparatively barbarous one the people cannot bear to have their follies ridiculed.

(18) No man is wholly admirable, but no man is a proper object of contempt; hence some beings who are not wholly admirable are not proper objects of contempt.

(19) If the object of education were to make pupils think rather than to accept certain conclusions, it would be conducted quite differently, there would be less rapidity of instruction and more discussion, more attempt to make education concern itself with matters in which the pupils feel some interest.

(20) Floods in the valley are due either to heavy rains or melting snow. There had been neither of these recently, so there will be no floods

CHAPTER VII

90. Describe the nature of Inductive Reasoning, and compare it with Deductive Reasoning.

91. " Inductive Reasoning is not limited to Science " Explain this statement. Why are examples of Induction usually taken from the various Sciences ?

92 " Science and common sense are not identical, if they were identical, what would be the value of Science ? " Critically examine this argument.

93. Distinguish between the *cause* and the *condition* (or conditions) of an event.

Distinguish also between *causal uniformities* and (*a*) uniformities of mere sequence, (*b*) uniformities of coexistence.

94. Does the fact that A has always preceded B in human experience justify us in concluding that A and B are causally connected ? Give full grounds for your answer.

95. On the basis of your answers to the two previous questions (93 and 94) carefully examine the meaning, or meanings, of the term " Law of Nature."

96. Why were men wrong in believing from innumerable examples of white swans, and none of black, that " all swans are white "; and right in believing that the melting point of lead being discovered in one case, all lead will prove to have the same melting point ?

97. " Not one of the methods of Direct Induction can get to work without a preliminary hypothesis " In what sense is this true ?

98 Compare and contrast the method of Single Agreement and the method of Double Agreement. Illustrate your answer.

99 What is the difference between Observation and Experiment ? Is it satisfactory to say that the investigator (*a*) is passive in the former and active in the latter, (*b*) uses apparatus in the latter but not in the former ?

100. Distinguish the method of Single Difference and the Joint Method of Difference and Agreement, and give scientific examples.

101. Explain what is meant by the testing of a hypothesis through negative instances.

102. Give an example of the method of Concomitant Variations, and formulate the rule or canon of the method.

103. Which of the methods of direct Induction do you consider to be most cogent, and why ?

104. How does the possibility of what is known as the " plurality of causes " constitute a difficulty for the scientific investigator ? Illustrate your answer by reference to the fact that there are many ways of " catching cold "

105. Given two substances, A and B, which have many characteristics in common, on what conditions should we be justified in assuming that because A has the property *m*, B has it also ?

106. What exactly is meant by "Analogy"?
How far does the argument from Analogy (*a*) resemble, (*b*) differ from, Induction by Incomplete Enumeration ("simple enumeration")?

107. Explain and illustrate the use of Hypothesis in Indirect Induction.

108. Give a concise statement of the conditions of a "good" or "legitimate" hypothesis. Illustrate your answer.

109. Write brief explanatory notes on the terms "Working Hypothesis"; "Barren Hypothesis"; "Negative Condition", "Negative Result"; "Fact"; "Theory"; "Empirical Law."

110. Distinguish between Verification and Proof, giving examples.

111. Enumerate the chief types of Explanation, giving a brief account and an example of each of them. What may be said in reference to the *limits* of scientific explanation?

112. "A sound method of investigation puts all intellects on a level." Carefully examine this.

113. "The spread of malarial fever is due to mosquitos" Give an outline of a mainly inductive argument intended to establish this proposition. If any deductive reasoning enters into the argument, point it out.

114. Show how the method of Single Difference could be applied to the effects of a law prohibiting the sale of intoxicating liquor, and criticise the application of the method in this case.

115. Three suggestions have been made to account for the immunity of the mongoose from snake-bite. They are: (*a*) As soon as it is bitten, the mongoose seeks a forest herb, eats it, and so is unharmed. (*b*) The rapidity of the movements of the mongoose baffles the snake. (*c*) The long hair of the mongoose is its protection from the fangs of the snake. Give an account of the methods that would be used to decide in favour of one or other of the hypotheses.

116. "Whenever we find arctic animals which, from whatever cause, do not require protection by the white colour, then neither the cold nor the snow-glare has any effect upon their coloration. The sable retains its rich brown fur throughout the Siberian winter; but it frequents trees at that season and not only feeds partially on fruits or seeds, but is able to catch birds among the branches of the fir-trees; with the bark of which its colour assimilates . . . But the most striking example is that of the common raven, which is a true arctic bird, and is found even in mid-winter as far north as any known bird or mammal. Yet it always retains its black coat, and the reason, from our point of view, is obvious. The raven is a

powerful bird and fears no enemy, while, being a carrion-feeder, it has no need for concealment in order to approach its prey. The colours of the sable and of the raven are, therefore, both inconsistent with any other theory than that the white colour of arctic animals has been acquired for concealment, and to that theory both afford a strong support." Analyse this passage so as to bring out the logical character of the argument through which the conclusion is reached.

117. "In worms the sense of smell apparently is confined to the perception of certain odours, and is feeble. They were quite indifferent to my breath so long as I breathed on them very gently. This was tried, because it appeared possible that they might thus be warned of the approach of an enemy They exhibited the same indifference to my breath whilst I chewed some tobacco, and while a pellet of cotton wool with a few drops of millefleurs perfume or of acetic acid was kept in my mouth. Pellets of cotton wool soaked in tobacco juice, in millefleurs perfume, and in paraffin, were held with pincers and were waved about within two or three inches of several worms, but they took no notice. On one or two occasions, however, when acetic acid had been placed on the pellets, the worms appeared a little uneasy, and this was probably due to the irritation of their skins. The perception of such unnatural odours would be of no service to worms; and as such timid creatures would almost certainly exhibit some signs of any new impression, we may conclude that they did not perceive these odours" Analyse this passage so as to bring out the logical character of the argument through which the conclusion is reached.

CHAPTER VIII

118. What are the chief causes of ambiguity in words? In what ways can ambiguity be avoided?

119. How would you distinguish between *logical* and *illogical* thinking?

120. Give a concise account of Bacon's view of the "Idola." What do you consider to be its value at the present time?

121. "The savage and the superstitious person are alike in having no sense of the causal connections of events" Show how an insufficient understanding of such connections results in belief in superstitions such as not sitting thirteen at a table, and so on

122. What precisely do you understand by a logical "Fallacy"? Distinguish "fallacy" and "false belief." Enumerate and concisely define the Fallacies incident to syllogistic (categorical and conditional) reasoning.

123. What are the most important kinds of error which may occur in scientific Observation? Illustrate your answer. Can these errors be regarded as mistakes in *reasoning?*

124. How far does the structure of Language throw light on the logical structure of Thought?

125. "Logic is not the same thing with knowledge, though the field of Logic is co-extensive with the field of knowledge." Critically discuss these statements.

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(*Terms explained in the Glossary are not mentioned in this Index*)

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